Data-driven modelling in dynamic networks

Paul M.J. Van den Hof

with Arne Dankers (Calgary) and Harm Weerts (TU/e)

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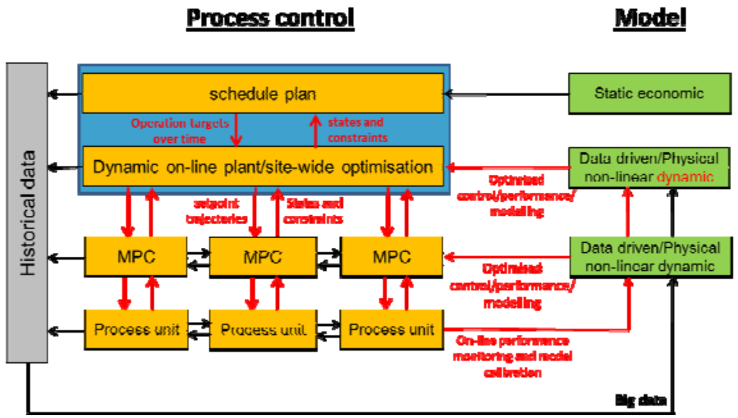
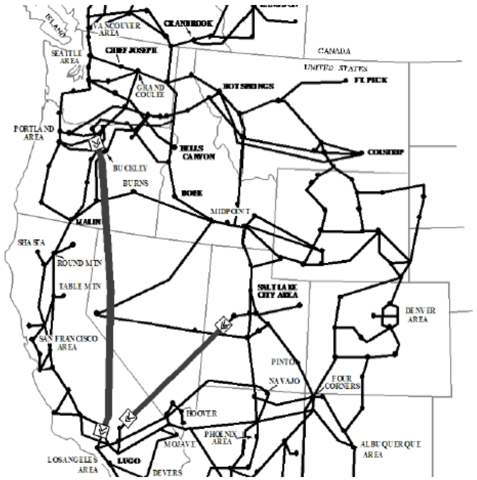
Where innovation starts

Introduction – dynamic networks

Decentralized process control

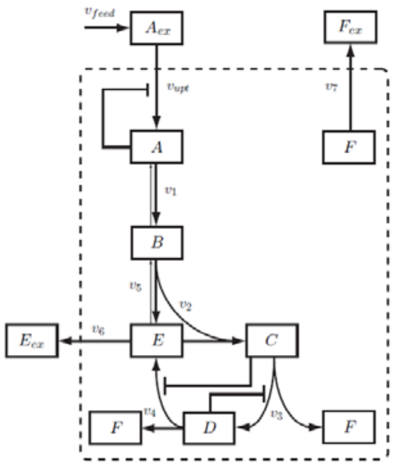


Power grid



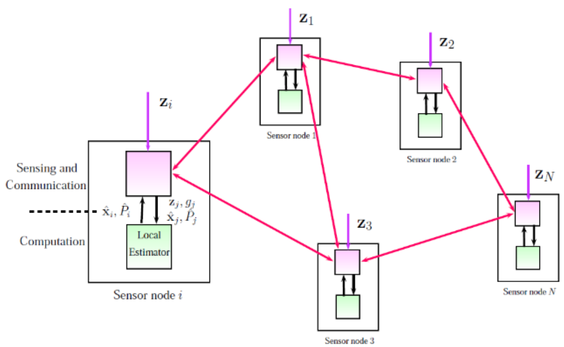
Pierre et al. (2012)

Metabolic network



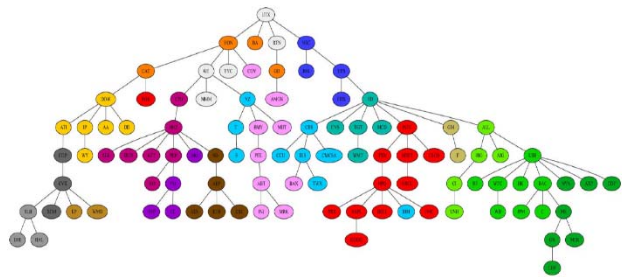
Hillen (2012)

Distributed control (robotic networks)



Simonetto (2012)

Stock market



Materassi et al. (2010)

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Introduction – dynamic networks

Drivers for **data-processing / data-analytics**

Providing the tools for **online**

- Model estimation / calibration / adaptation

to accurate perform online model-based **X:**

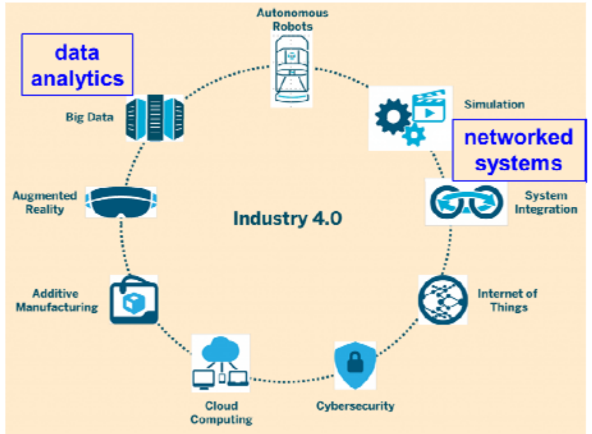
- Monitoring
- Diagnosis and fault detection
- Control and optimization
- Predictive maintenance
- Controller reconfiguration
- ....

Turn large amounts of (relatively inexpensive) data into process/economic value

Industry 4.0 – process operations aspects

From isolated (statically) optimized units to

- integrated chains/networks of production units,
- fully automated, high level of sensing/actuation,
- data and product flows across classical (company) borders (suppliers,customers, energy grid)
- modular build-up
- continuously monitored for control, optimization, (predictive) maintenance, analysis, ...
- adapting to changing circumstances (process and market conditons), and learning
- economically optimized
- supervised by new-generation HMI technology and operators



[Boston Consulting Group report: "Industry 4.0, The Future of Production & Growth in Manufacturing Industries", 2015]

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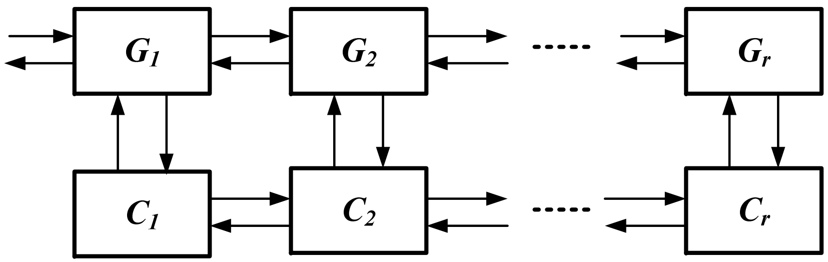
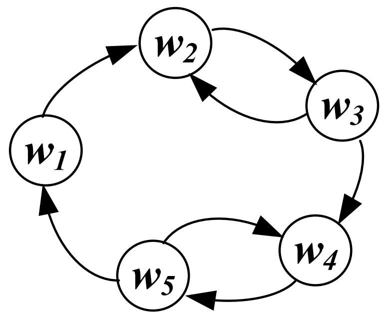
Introduction – dynamic networks

Dynamical systems are considered to have a more complex structure:

distributed control system (1d-cascade)



dynamic network



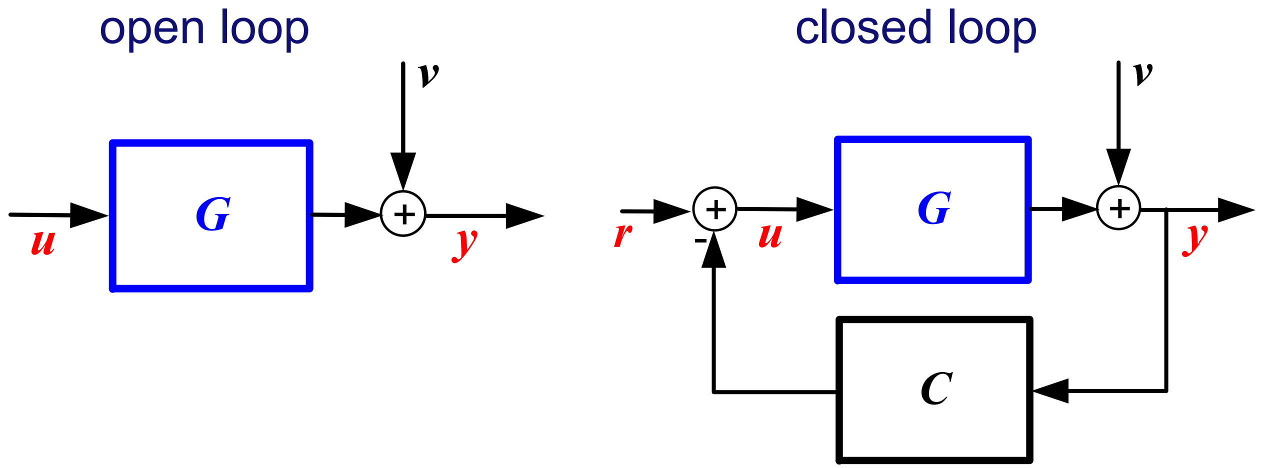
(distributed MPC, multi-agent systems, biological networks, smart grids,…..)

For on-line monitoring / control / diagnosis it is attractive to be able to ***identify***

- (changing) dynamics of modules in the network
- (changing) interconnection structure

Introduction - identification

The classical (multivariable) identification problems: [Ljung (1999)]

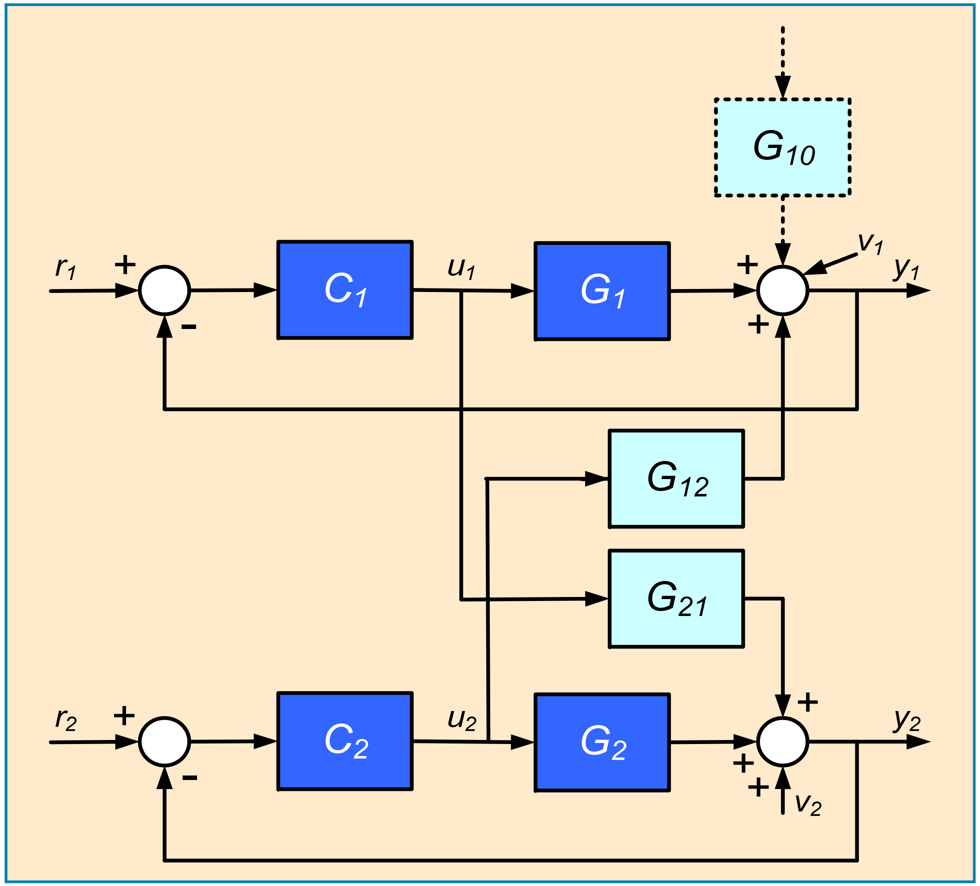


Identify a plant model $\hat{G}$ on the basis of measured signals *u, y* (and possibly *r*)

- **We have to move from fixed and known configuration to deal with and exploit *structure* in the problem.**

Introduction - identification

Example decentralized MPC; 2 interconnected MPC loops



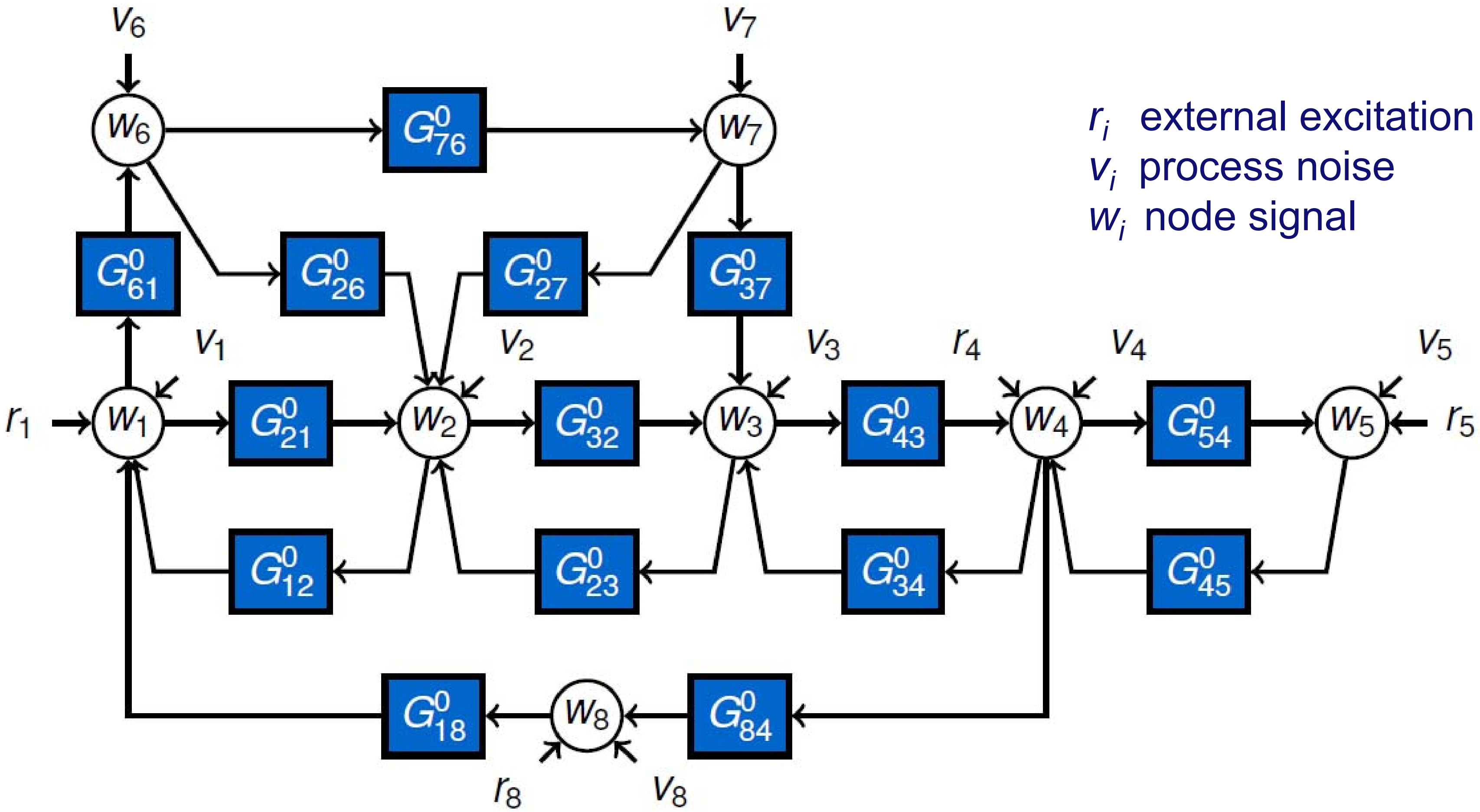
Target:
Identify interaction dynamics

$$G_{21}, G_{12}$$

Addressed by
Gudi & Rawlings (2006)
for the situation $G_{12} = 0$
(no cycles)

Gudi, R. D. and Rawlings, J. B. (2006). Identification for decentralized model predictive control. AIChE Journal, 52(6):2198-2210.

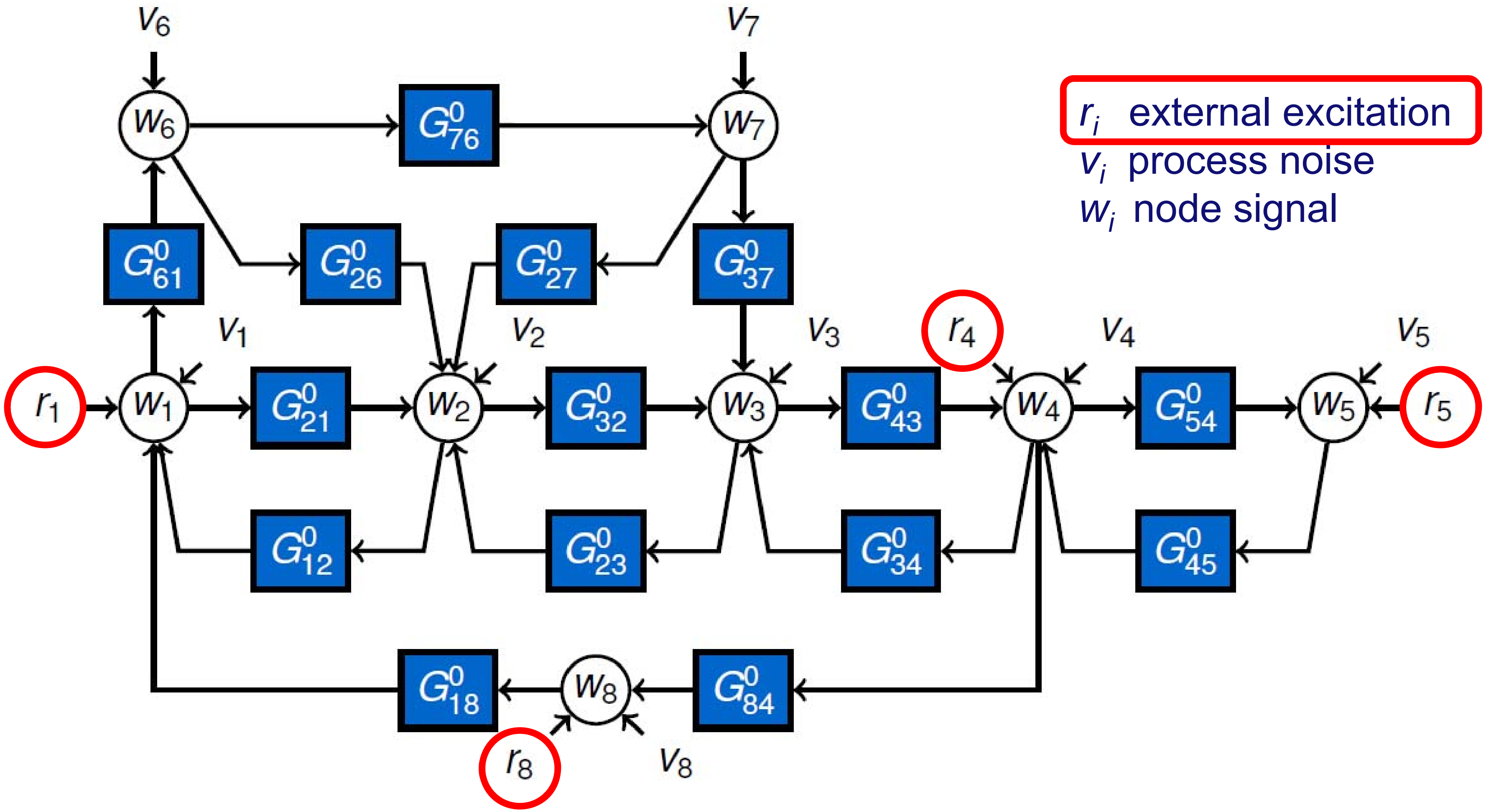
Introduction – Dynamic network identification



r_i external excitation
v_i process noise
w_i node signal

Some modules may be known (e.g. controllers)

Introduction – Dynamic network identification



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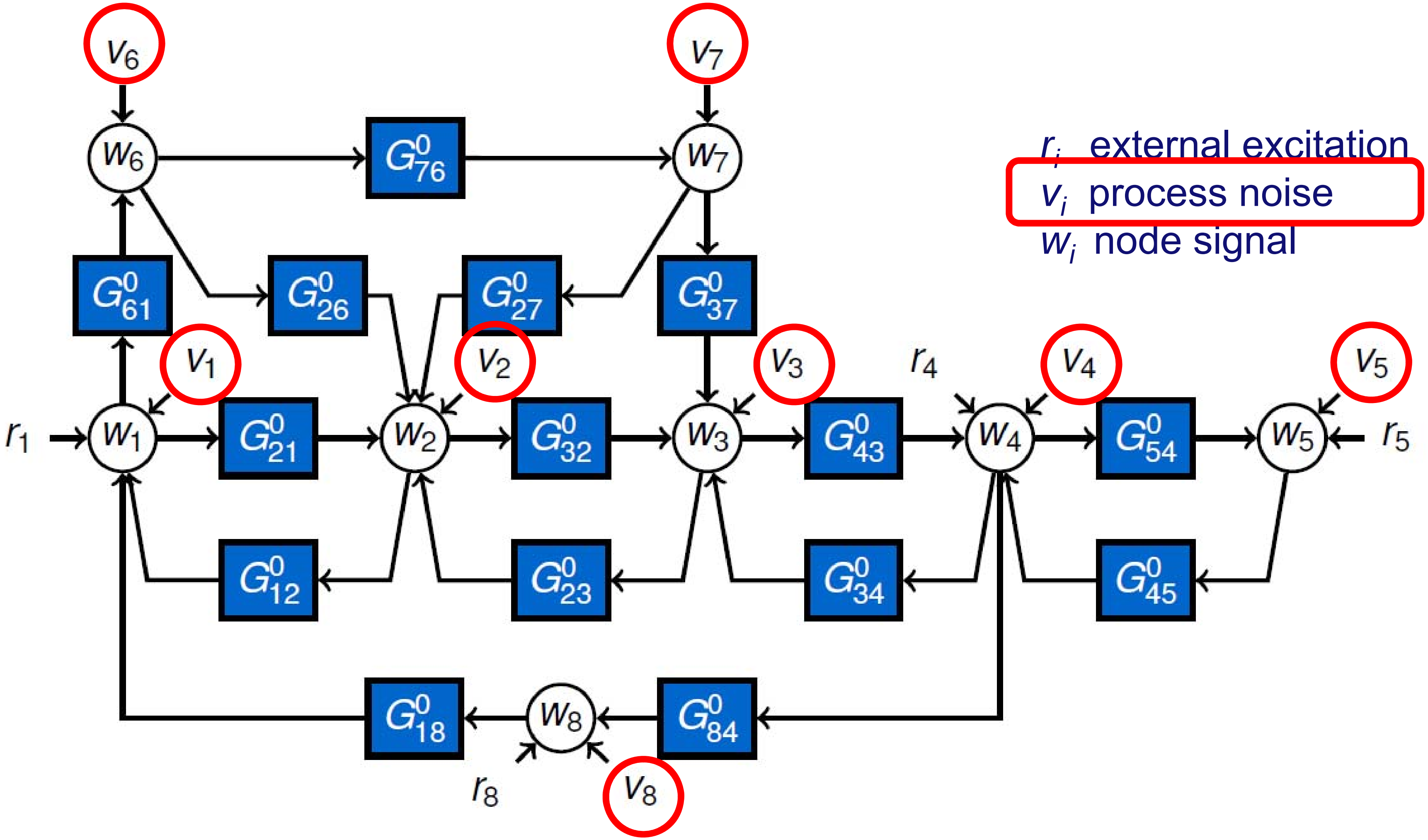

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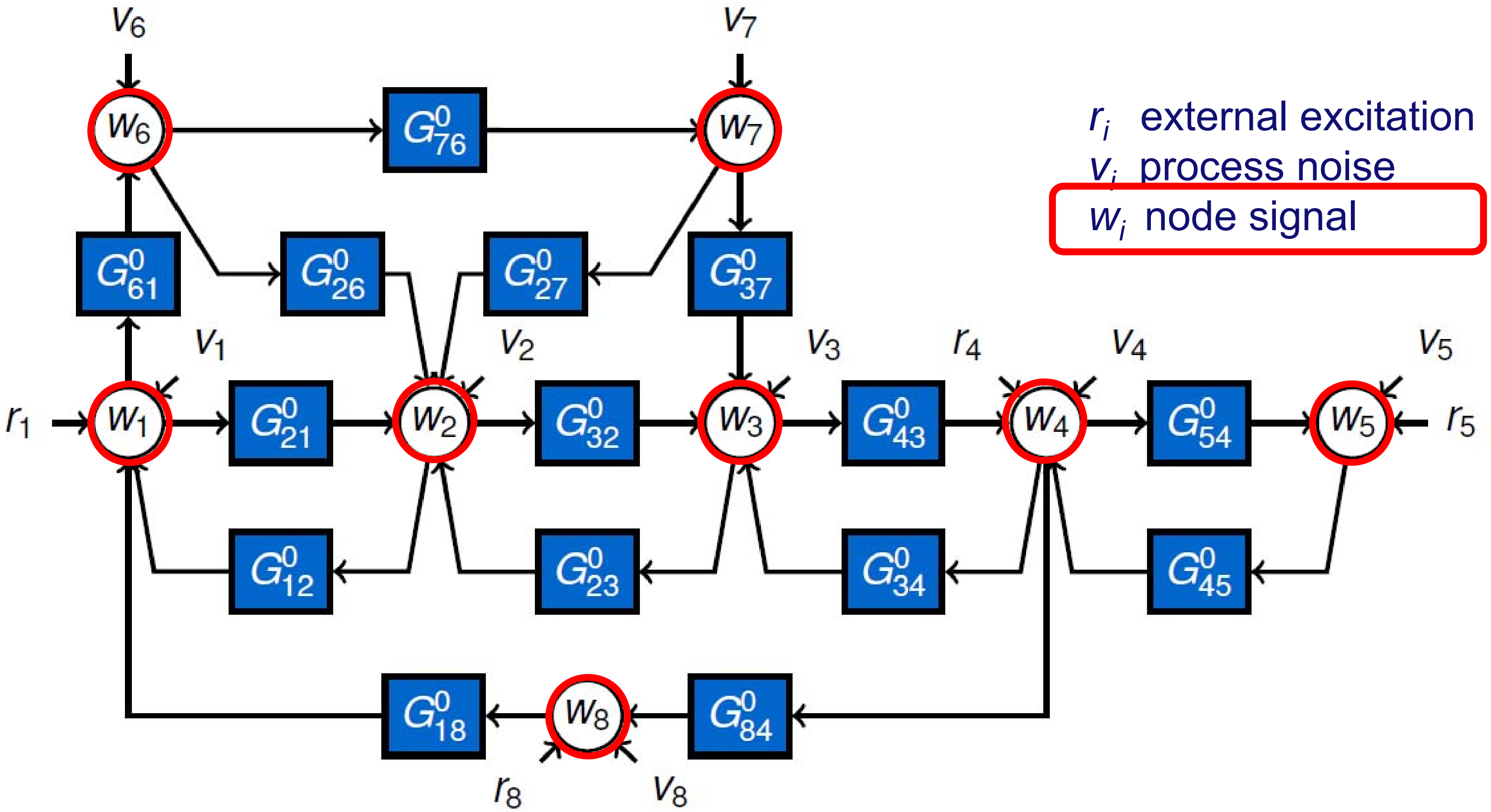
Introduction – Dynamic network identification



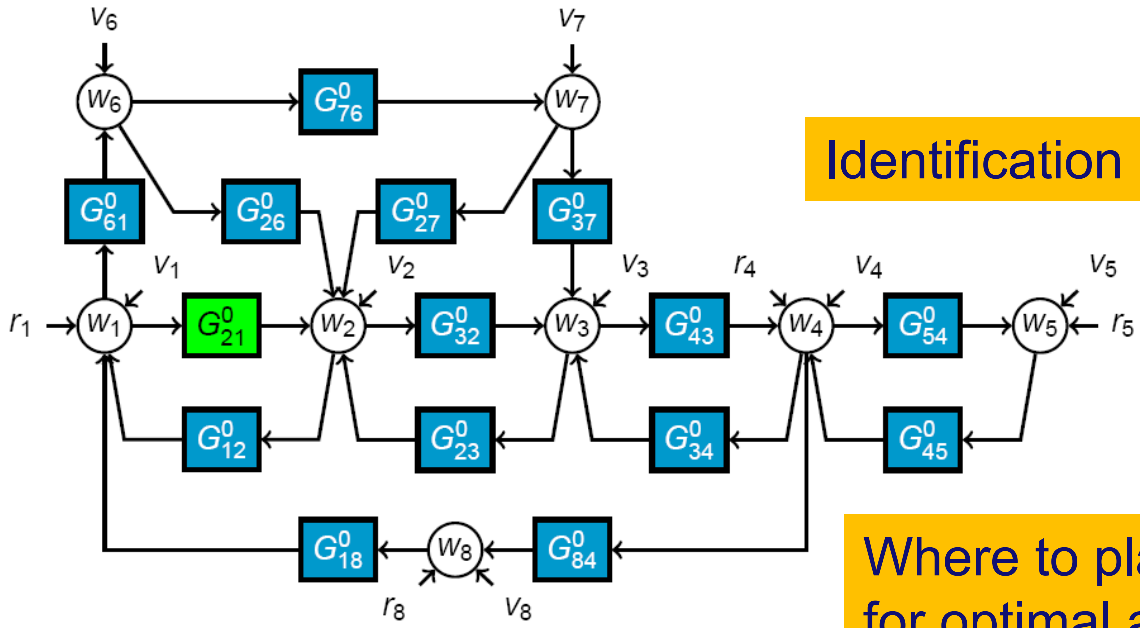
r_i external excitation

v_i process noise

w_i node signal

Some modules may be known (e.g. controllers)

Introduction – relevant identification questions



Identification of a single (local) module?

Where to place sensors and actuators for optimal accuracy?

How to utilize known structure/topology and known modules?

Can we identify the topology?

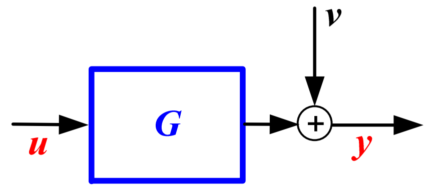
Is the full network identifiable?

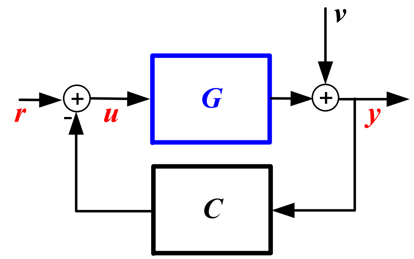
[P.M.J. Van den Hof, A. Dankers, P.S.C. Heuberger and X. Bombois. *Automatica*, October 2013]

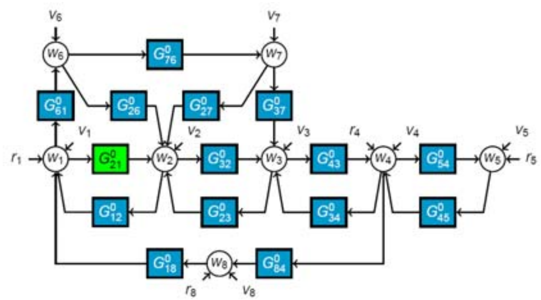
Contents

Towards dynamic network identification

- **Basic identification tools: direct and projection**
  - **From closed-loop to dynamic networks**
- **Single module identification - consistency**
  - **full MISO models**
  - **predictor input selection**
- **Example of decentralized control**
- **Additional results and discussion**







Methods for closed-loop identification

1. Direct method

Relying on full-order noise modelling;
Prediction error

$$\varepsilon(t,\theta) = H(\theta)^{-1}[y(t) - G(\theta)u(t)]$$

Using only signals *u* and *y*, discarding *r*

$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N}\sum_{t=1}^{N}\varepsilon(t,\theta)^2$$

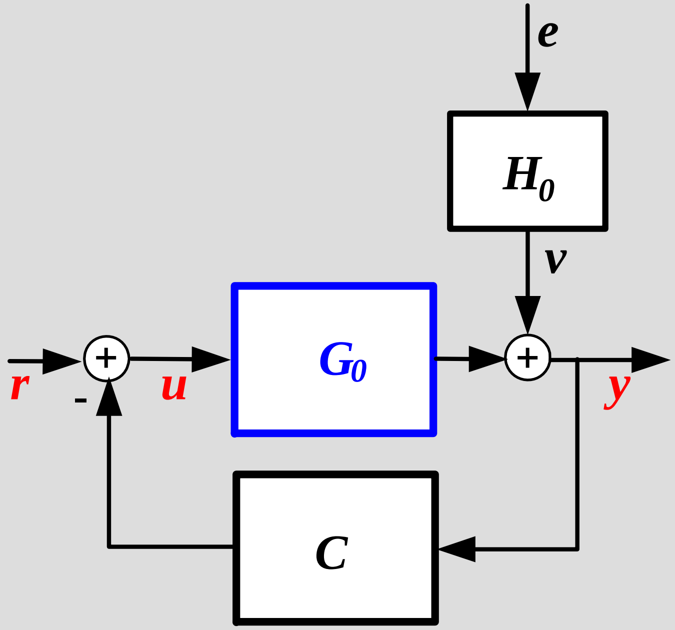
2. Projection/two-stage/IV method

Relying on measured external excitation r

$$\varepsilon(t,\theta) = H(\rho)^{-1}[y(t) - G(\theta)u^r(t)]$$

with u^r the signal u projected onto r

Similar least squares criterion.



Plant representation

$$y(t) = G_0u(t) + H_0e(t)$$

e white noise

r and v uncorrelated

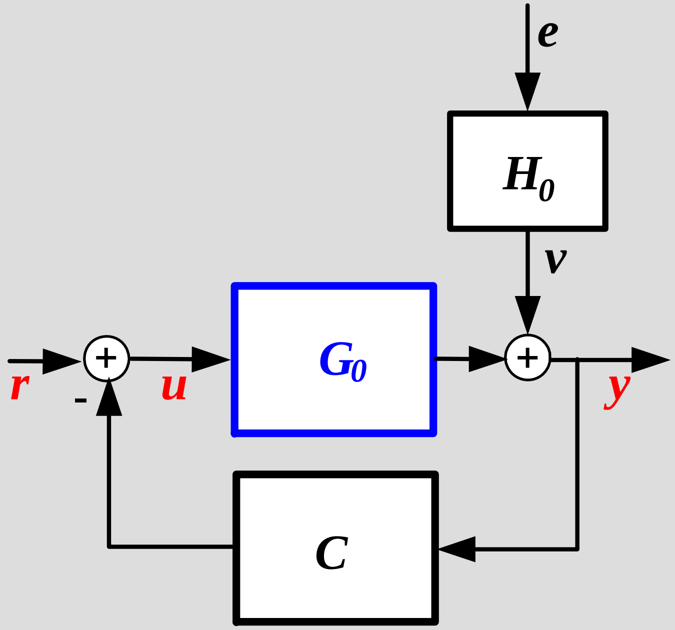
Methods for closed-loop identification

1. Direct method [Ljung, 1987]

Consistent estimate of $\{G_0, H_0\}$
provided that *u* is sufficiently exciting

2. Projection/two-stage/IV method [Van den Hof & Schrama, 1993]

Consistent estimate of G_0
provided that u^r is sufficiently exciting



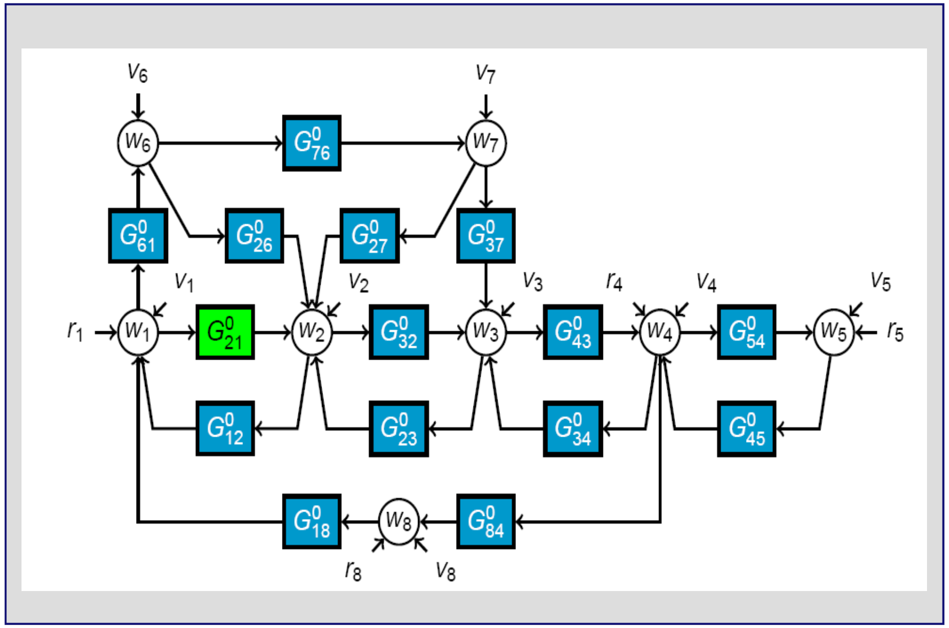
Plant representation

$$y(t) = G_0 u(t) + H_0 e(t)$$

e white noise

r and v uncorrelated

Network Setup



Assumptions:

- Total of L nodes
- Network is well-posed
  $I - G^0$ causally invertible
- Stable (all signals bounded)
- All $w_m, m = 1, \cdots L,$ measured, as well as all present r_m
- Modules may be unstable

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_L \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

Identifying a module

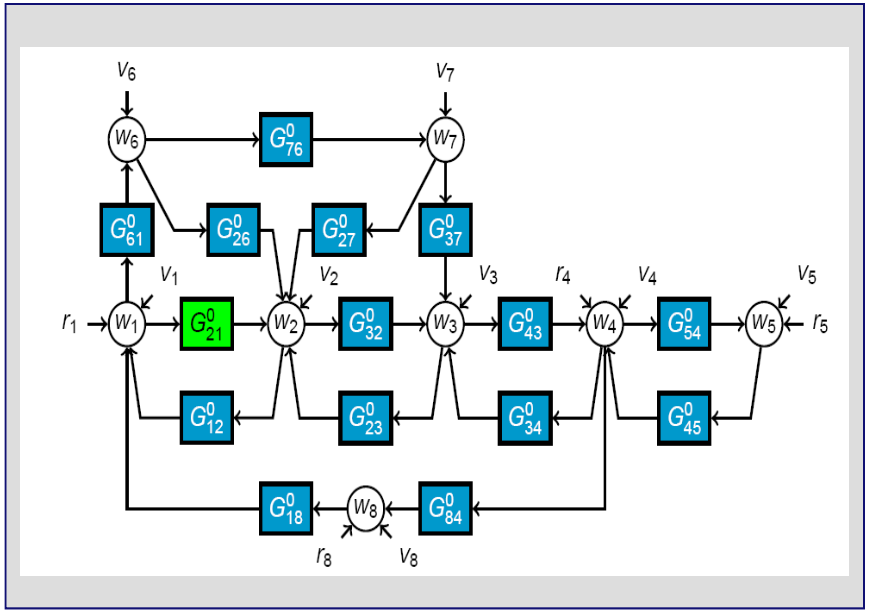
Options for identifying a module:

- **Identify the full MIMO system:**

$$w = (I - G^0)^{-1}[r + v]$$

from measured r and w.

Global approach with “standard” tools



- **Identify a local (set of) module(s) from a (sub)set of measured r_k and w_ℓ**

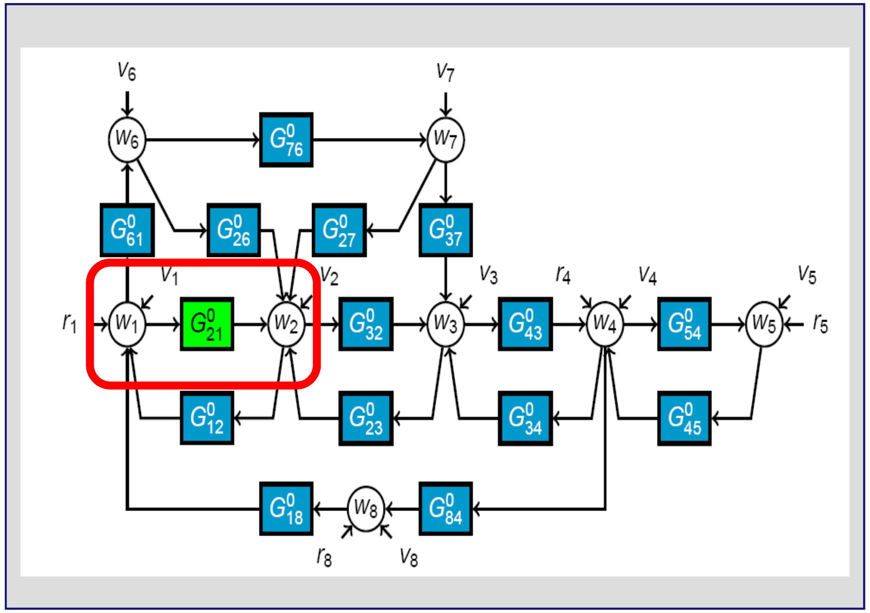
Local approach with “new” tools and structural conditions

Identifying a module

How to identify a module:

Suppose we are interested in G_{21}^0

Can it be identified from measured input w_1 and output w_2?



➡ Typically bias will occur due to "neglecting" the rest of the network

- Non-modelled disturbances on w_2 can create problems
- The observed transfer between w_1 and w_2 is not necessarily G_{21}^0

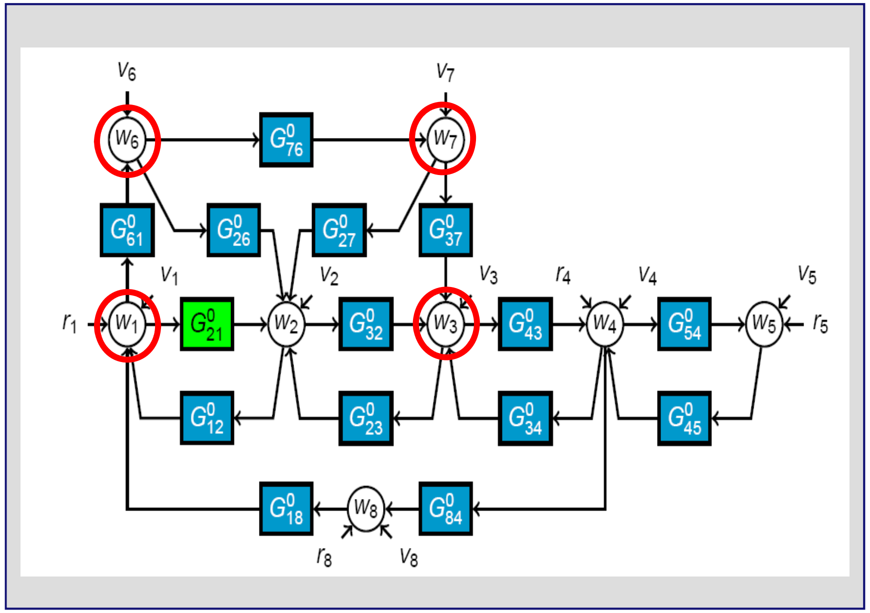
Identifying a module

How to identify a module:

Two approaches for finding G^0_{21}

- **Full MISO approach:**
  Include all node signals that directly map into w_2 in an input vector, and identify a MISO model

- **Predictor input selection:**
  Formulate conditions for checking the sufficiency of set of nodes to include as inputs in a MISO model



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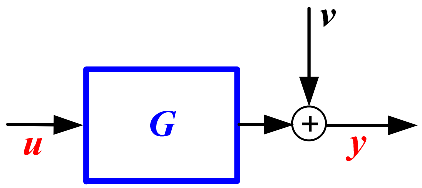

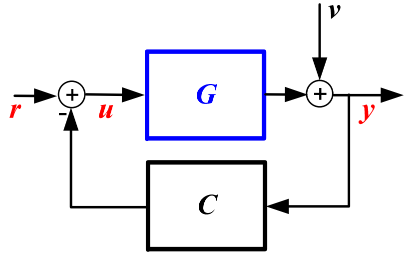

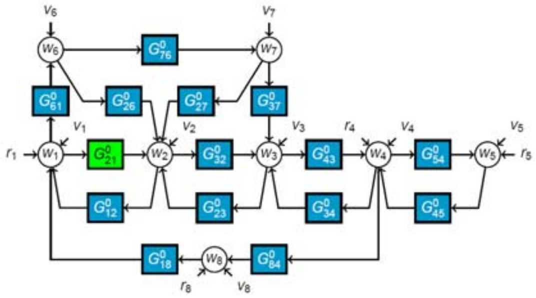

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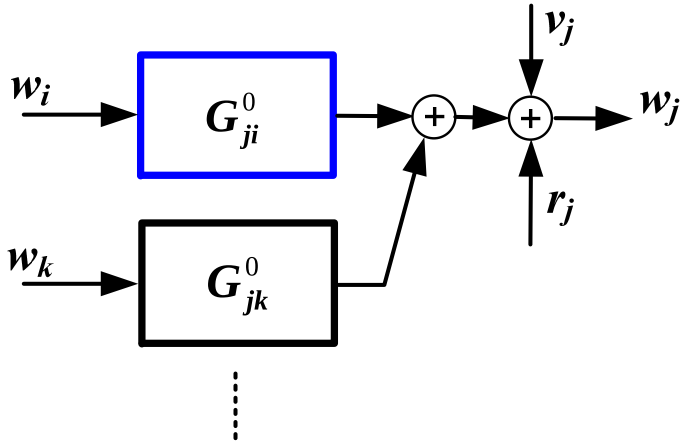
Full MISO models – Direct method



- Module of interest: G^0_{ji}
- Separate the modules G^0_{jk} into

  known modules: $G^0_{jk},\ k \in \mathcal{K}_j$

  and **unknown** modules: $G^0_{jk},\ k \in \mathcal{U}_j$
- Determine: $\bar{w}_j(t) = w_j(t) - r_j(t) - \sum_{k\in\mathcal{K}_j} G^0_{jk}(q)w_k(t)$
- Prediction error: $\varepsilon(t,\theta) = H_j(\theta)^{-1}[\bar{w}_j(t) - \sum_{k\in\mathcal{U}_j} G_{jk}(\theta)w_k(t)]$

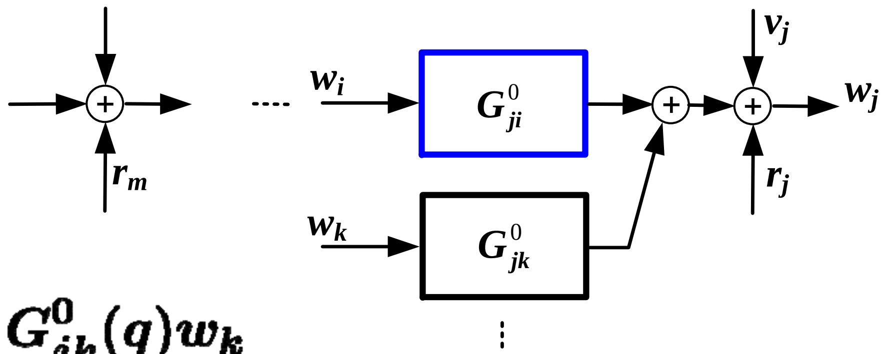
→ **Simultaneous identification of $G^0_{jk},\ k \in \mathcal{U}_j$ and H^0_j**

→ **Consistent estimates if $\{w_k\}_{k\in\mathcal{U}_j}$ sufficiently exciting, and $\Phi_v(\omega)$ diagonal**

[P.M.J. Van den Hof et al., *Automatica*, October 2013]

Network Identification – Projection method

Algorithm:



- Find an r_m with a path to w_i such that $w_i^{r_m}$ is present
- Construct: $$\bar{w}_j = w_j - r_j - \sum_{k \in \mathcal{K}_j} G^0_{jk}(q) w_k$$
  (known terms)
- Prediction error: $$\varepsilon(t,\theta) = H_j(\rho)^{-1}[\bar{w}_j - \sum_{k \in \mathcal{U}_{is}} G_{jk}(\theta) w_k^{r_m}]$$

  where all inputs $k \in \mathcal{U}_{is} \subset \mathcal{U}_j$ are considered that are correlated to r_m

Consistent identification of G^0_{jk}, $k \in \mathcal{U}_{is}$ provided that $\{w_k^{r_m}\}_{k \in \mathcal{U}_{is}}$ sufficiently exciting

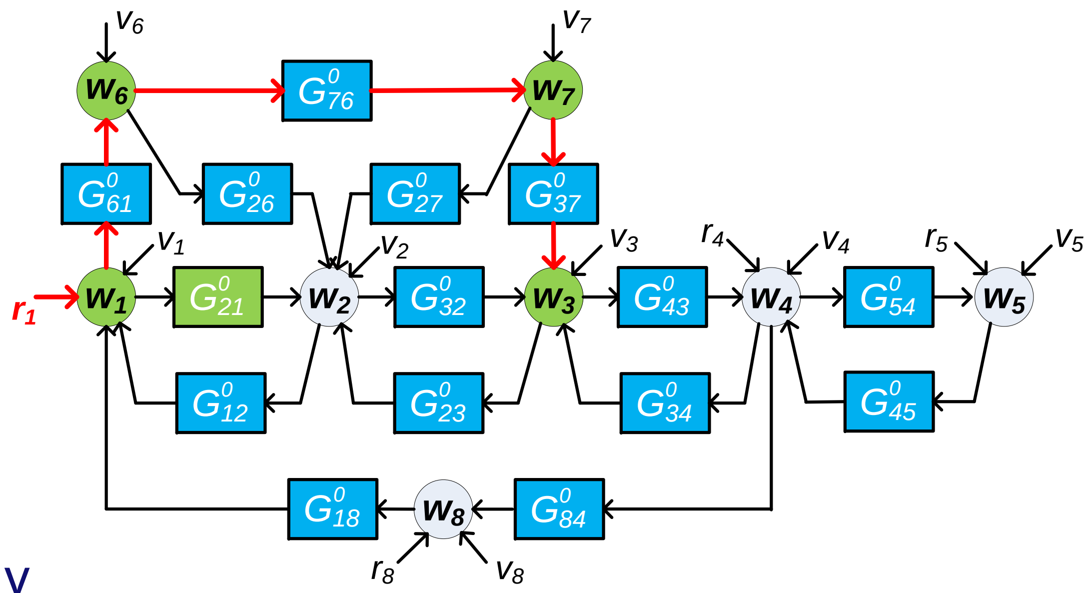
- This extends to multiple signals r_m

[P.M.J. Van den Hof et al., *Automatica*, October 2013]

Network Identification – Two-stage method

Example

- External signal r_1
- Input nodes to w_2 that are correlated with r_1 : w_1, w_6, w_7, w_3
- So 4 input, 1 output problem
- Projected inputs will generally not be sufficiently exciting (we need 4 independent sources)



- Include r_4, r_5 and r_8 as external signals
- Input nodes remain the same as for direct method

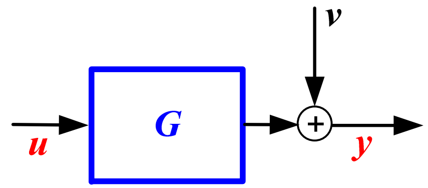
Network Identification – Full MISO models

Observations:

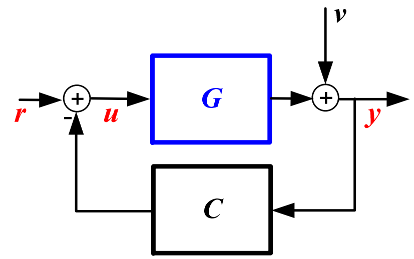
- Consistent identification of single transfers is possible, dependent on network topology and reference excitation
- Choice between estimating accurate noise models (direct method) and utilizing reference excitation (projection method)
- Excitation conditions on (projected) input signals can be limiting
- Network topology conditions on r_m can simply be checked by tools from graph theory

Contents

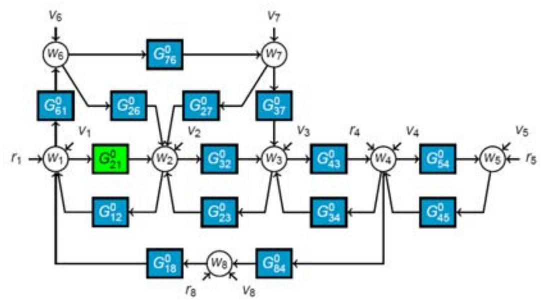
Towards dynamic network identification



- Basic identification tools: direct and projection
  - From closed-loop to dynamic networks



- Single module identification - consistency
  - full MISO models
  - predictor input selection



- Example of decentralized control
- Additional results and discussion

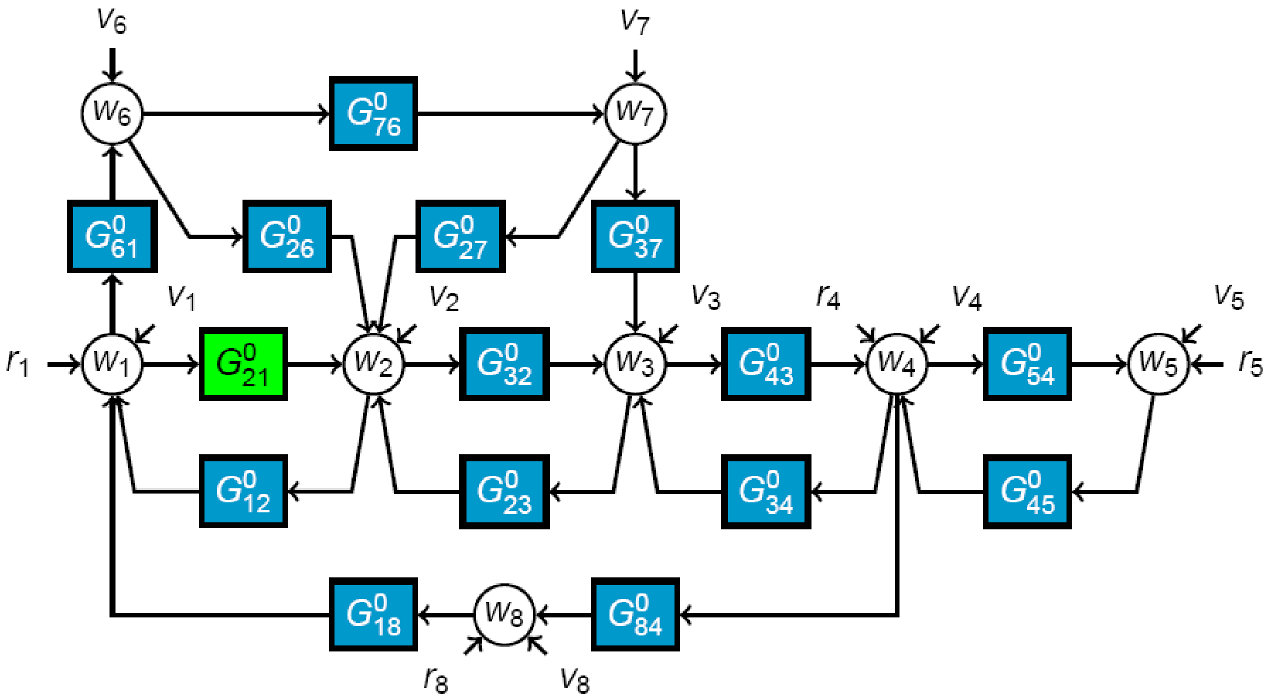
Predictor input selection

- So far: predictor input choice not very flexible
- What if some signals are hard (expensive) to measure?
- What if we would like to have flexibility in placing sensors?
- Can we formulate (more relaxed) conditions on nodes to be measured, for allowing a consistent module estimate?

Predictor input selection

There are two basic mechanisms that “deteriorate” the transfer G^0_{ji} when nodes are removed:

1. **Parallel paths**
2. **Loops around w_j**



To maintain G^0_{ji} these should be “blocked” by measured nodes (predictor inputs)

Predictor input selection: condition 1 and 2

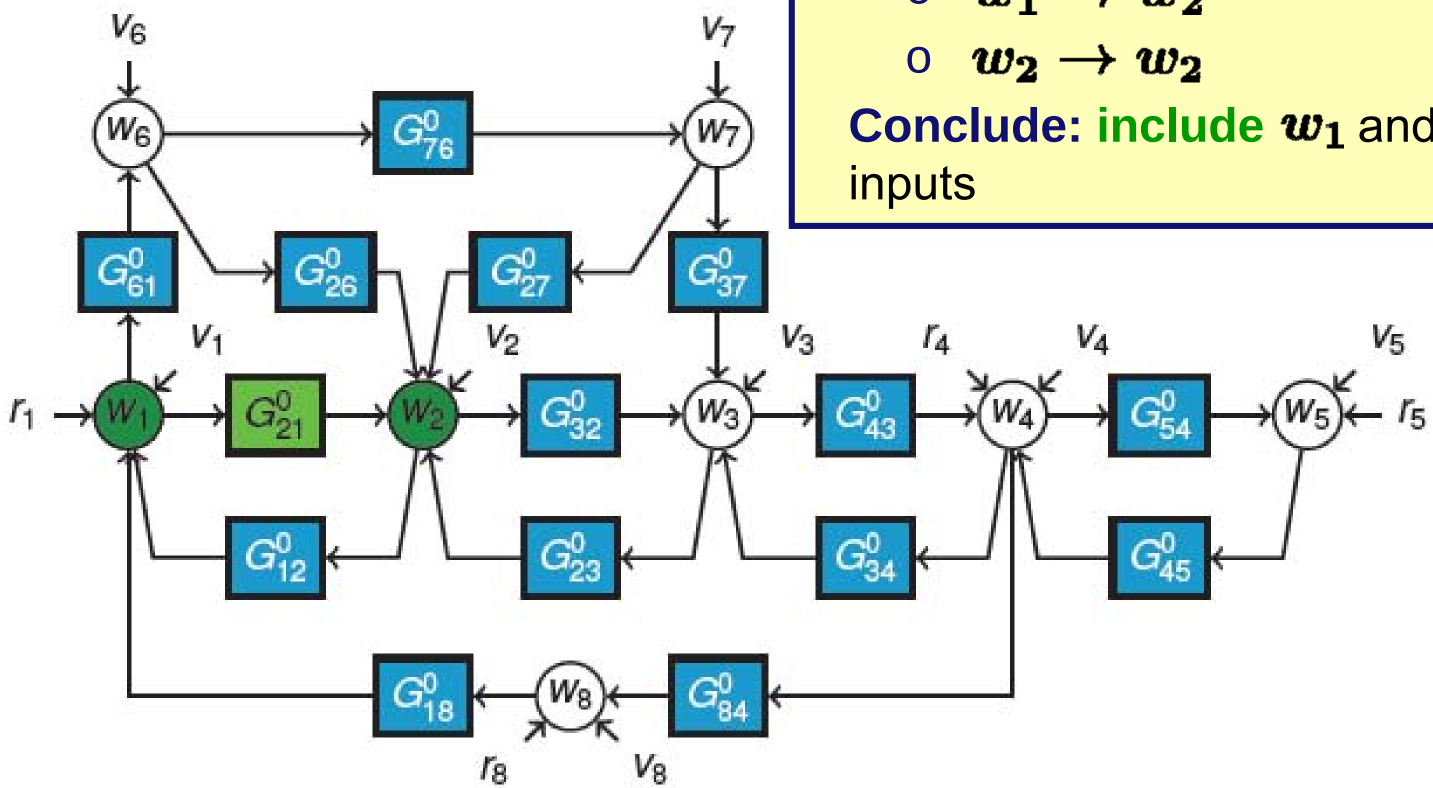
Objective: obtain an estimate of G^0_{ji}

Consistent estimates of G^0_{ji} are possible if:

1. w_i is included as predictor input
2. Each parallel path from $w_i \rightarrow w_j$ passes through a node chosen as predictor input
3. Each loop from $w_j \rightarrow w_j$ passes through a node chosen as predictor input

Example with predictor input conditions

Objective: Estimate G^0_{21}.



Conditions: Include variable on every path

- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

Conclude: include w_1 and **…** as predictor inputs

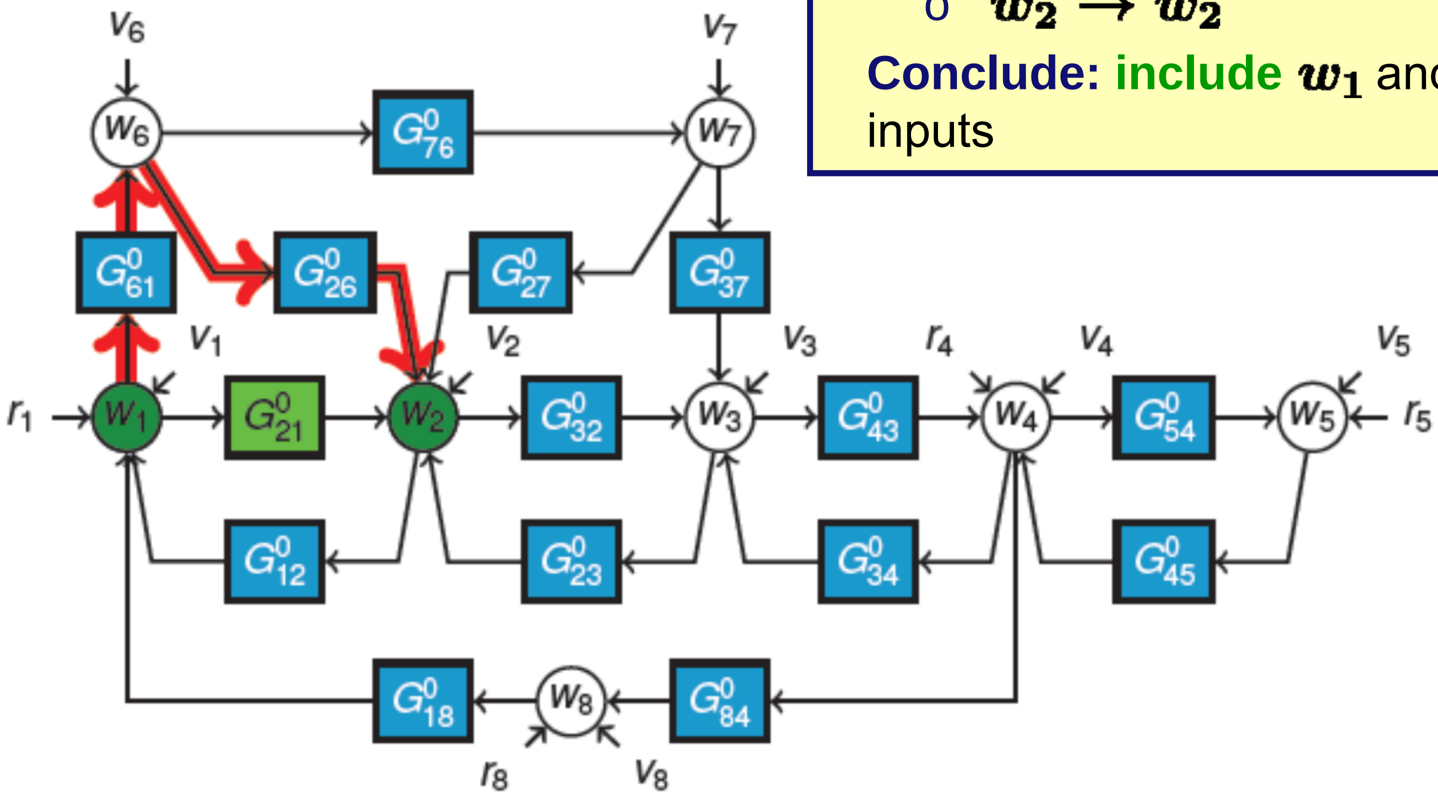
Example with predictor input conditions

Objective: Estimate G_{21}^0.

Conditions: Include variable on every path

- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

Conclude: include w_1 and … as predictor inputs



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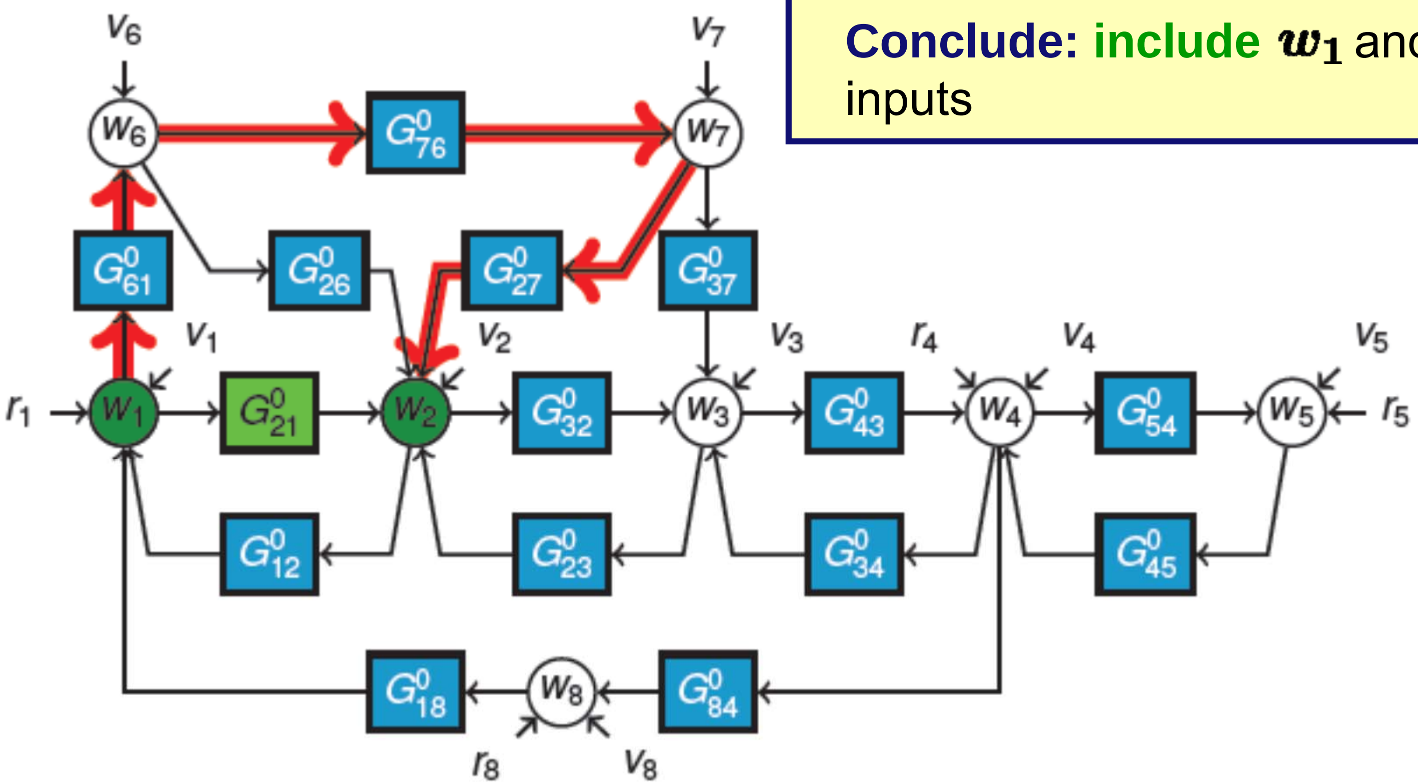

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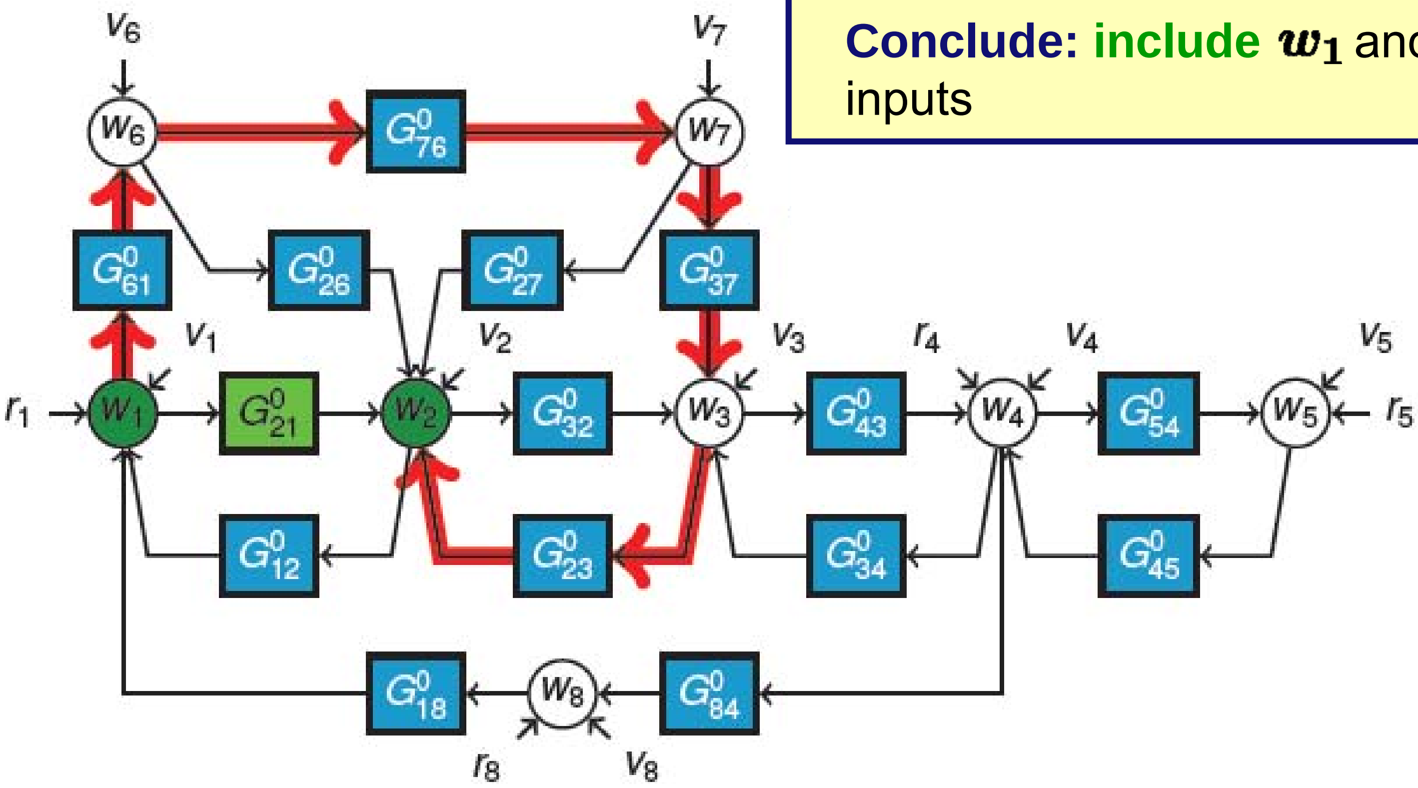

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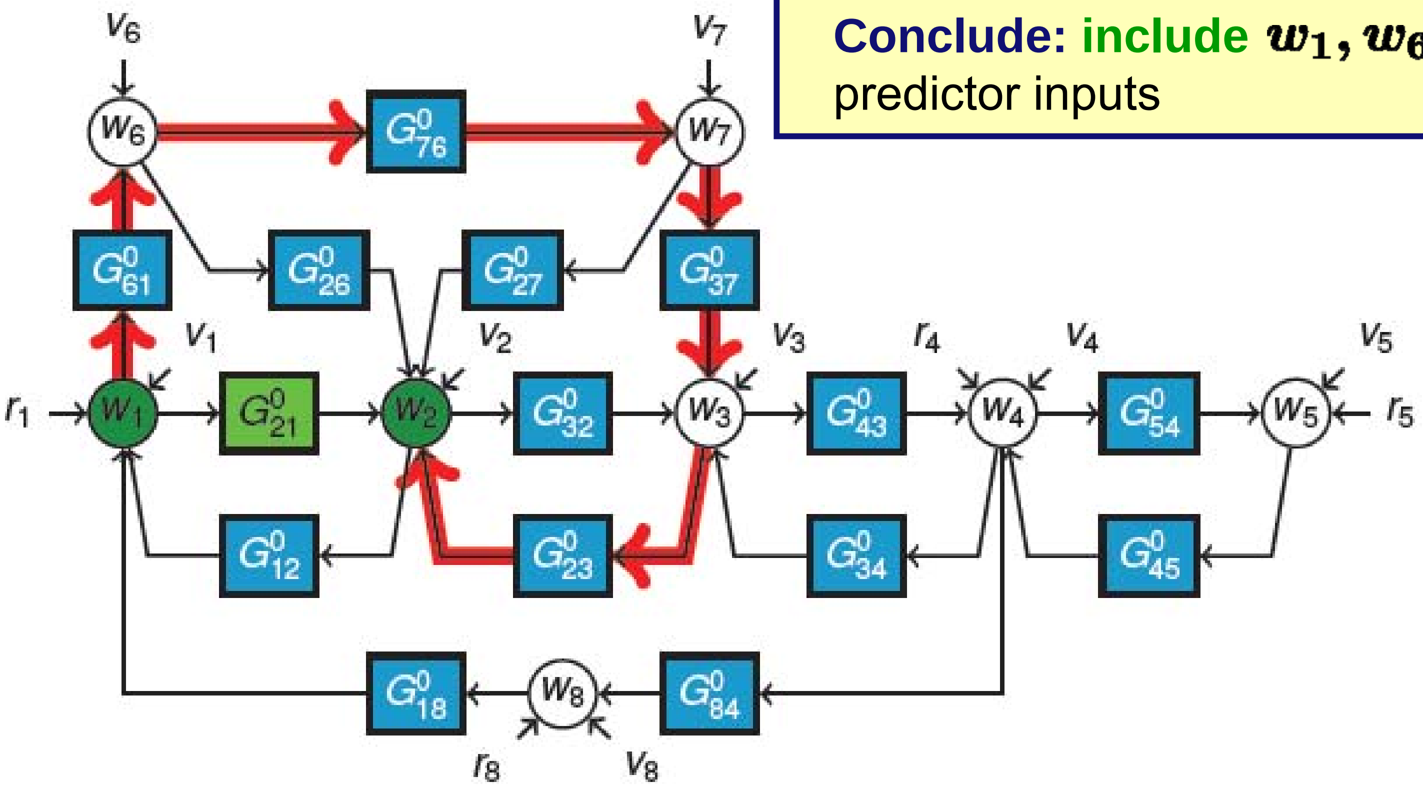


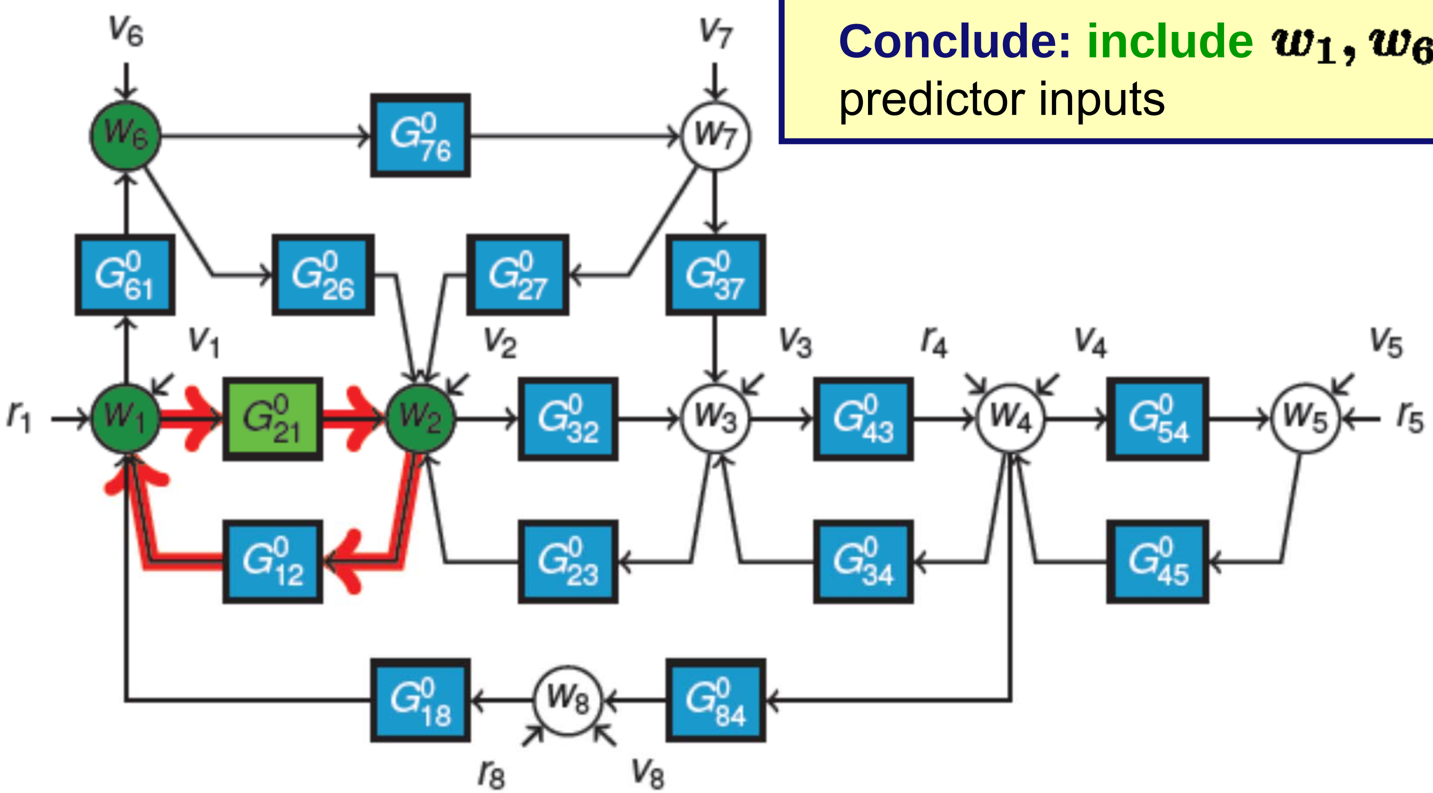
Conditions: Include variable on every path

- $w_1 \rightarrow w_2$ ➡ **Include w_6 in predictor**
- $w_2 \rightarrow w_2$

Conclude: include w_1, w_6 and **…** as predictor inputs

Example with predictor input conditions

Objective: Estimate G_{21}^0.



Conditions: Include variable on every path

- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

Conclude: include w_1, w_6 and **…** as predictor inputs

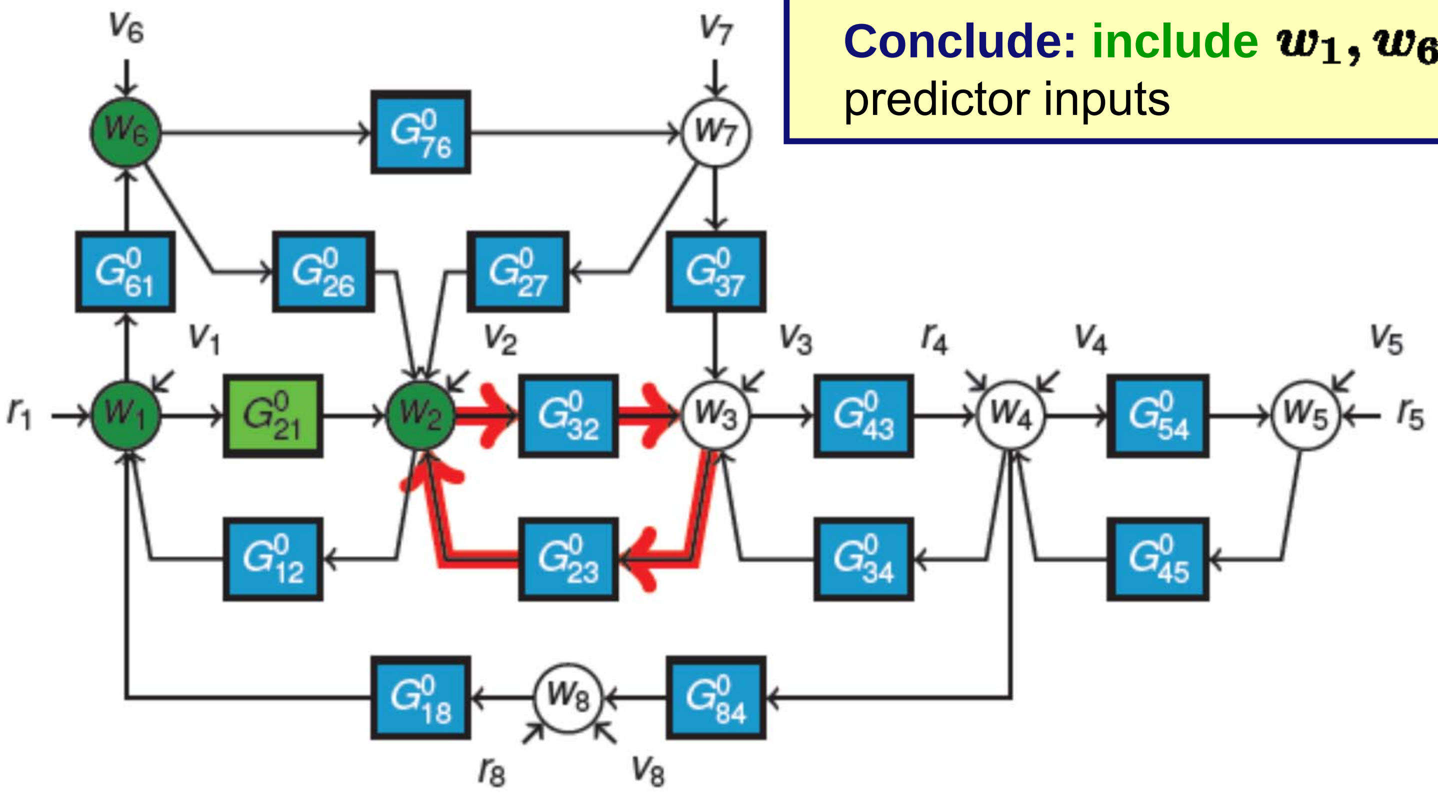
Example with predictor input conditions

Objective: Estimate G_{21}^0.

Conditions: Include variable on every path

- $w_1 \rightarrow w_2$
- $w_2 \rightarrow w_2$

Conclude: include w_1, w_6 and **…** as predictor inputs



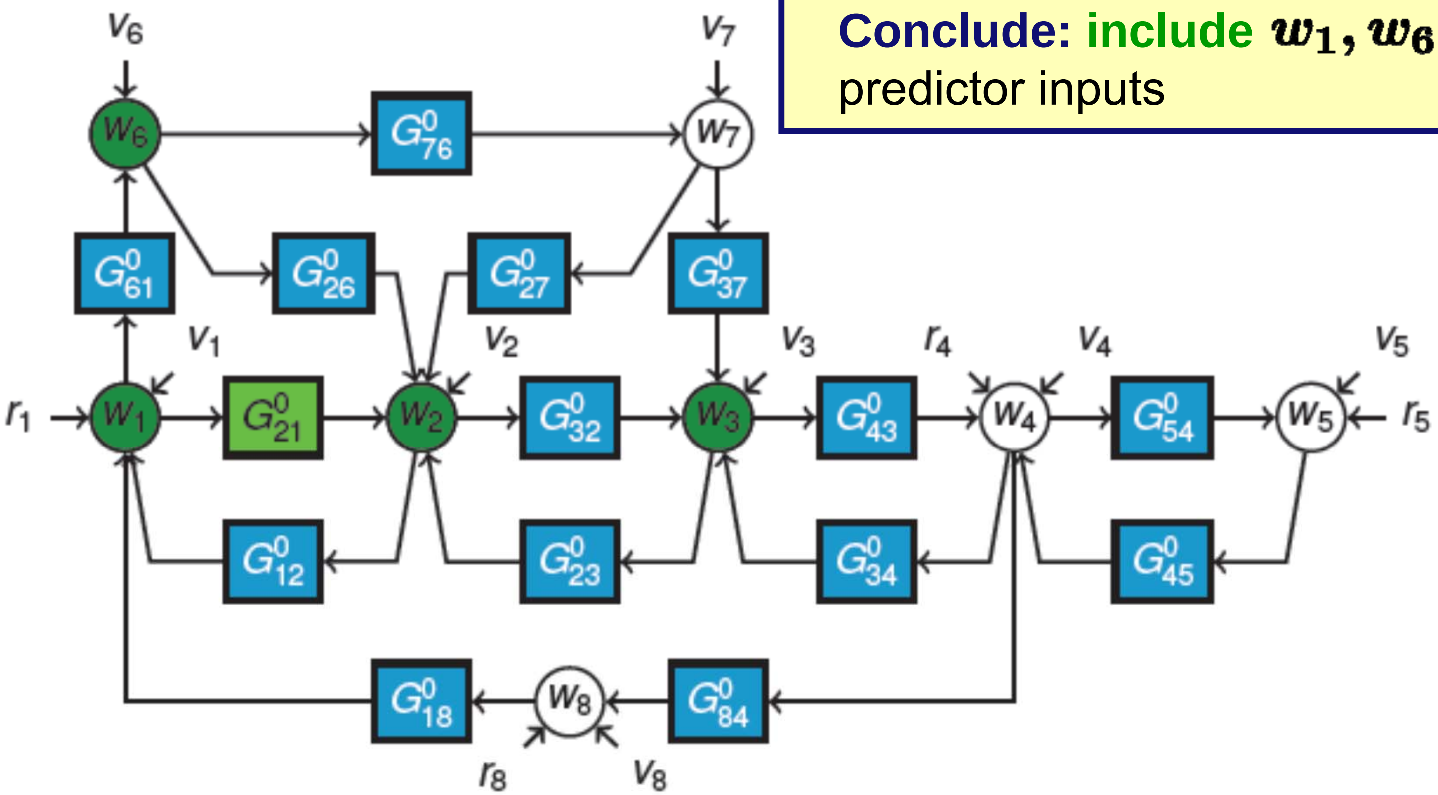
Example with predictor input conditions

Objective: Estimate G^0_{21}.

Conditions: Include variable on every path

- $w_1 \rightarrow w_2$ ➡ **Include w_6 in predictor**
- $w_2 \rightarrow w_2$ ➡ **Include w_3 in predictor**

Conclude: include w_1, w_6 and w_3 as predictor inputs



Predictor input selection

Result

The consistency results of both direct and projection method remain valid if

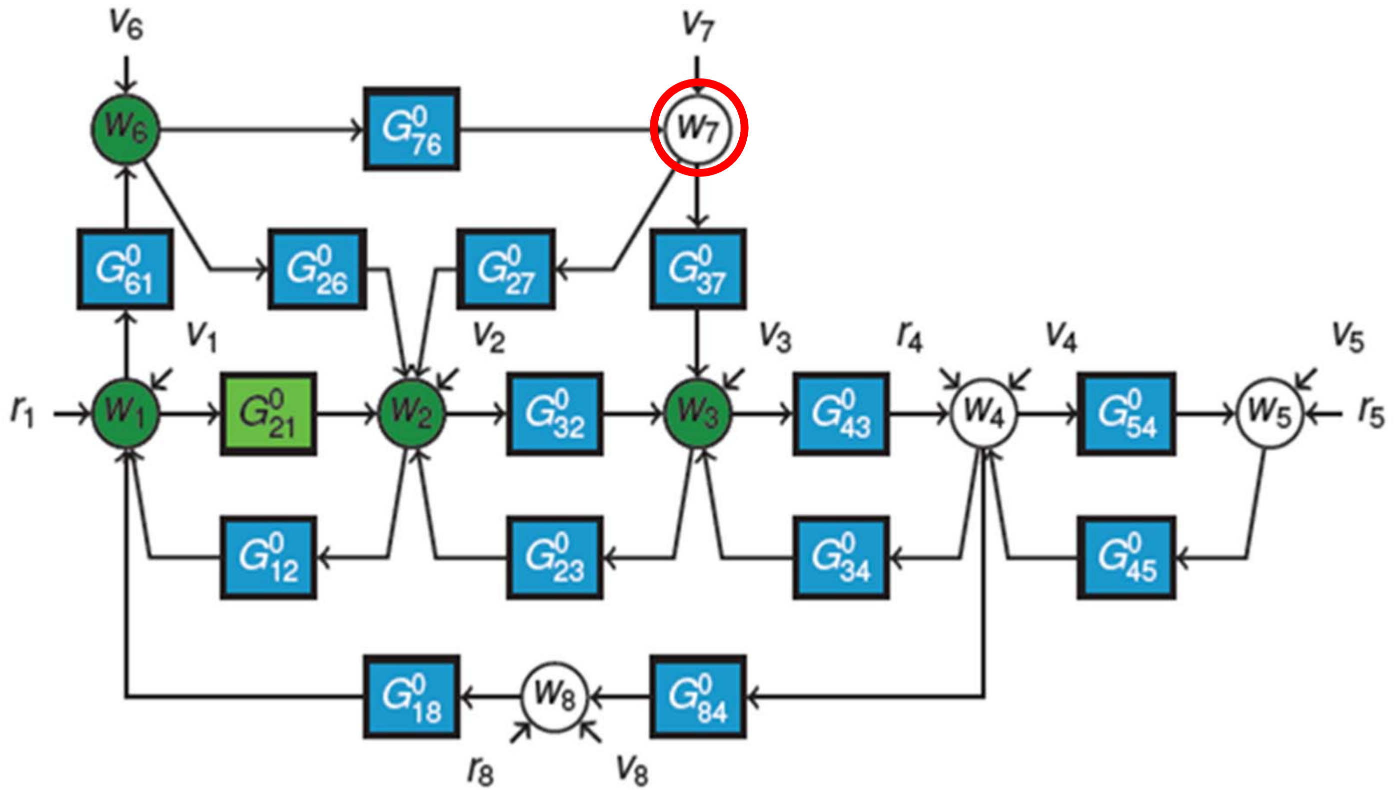
- the set $\mathcal{D}_j$ of predictor inputs satisfies the formulated conditions
- For the direct method: there are no **confounding variables**
- For the projection method: no excitation signal used for projection, has a path to w_j that does not pass through a node in $\mathcal{D}_j$

In the "full" MISO case: consistent estimates of all G^0_{jk}, $k \in \mathcal{U}_j$

In the "selected" predictor input case: consistent estimates of G^0_{ji}

[A. Dankers et al., Identification of dynamic models in complex networks with prediction error methods – predictor input selection. *IEEE Trans. Automatic Control*, 61(4), 937-952, april 2016.]

Predictor input selection



For direct method: w_7 is a *confounding variable* and needs to be included

For projection method: no problems

Immersed network

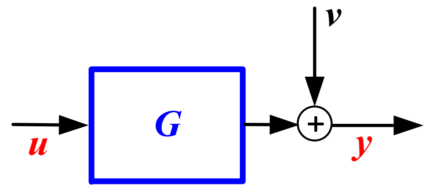
- The two conditions (parallel paths and loops on output) result from an analysis of the so-called **immersed network**
- The **immersed network** is constructed on the basis of a reduced number of node variables only, and leaves present node signals invariant
- Whether dynamics in the **immersed network** is invariant can be verified with the graph theory/tools of separating sets.

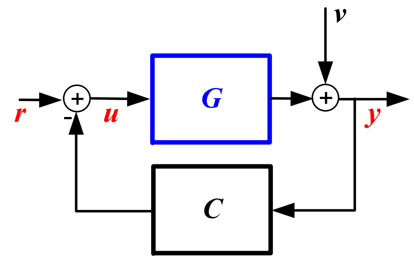
[A. Dankers, P.M.J. Van den Hof, P.S.C. Heuberger and X. Bombois. Identification of dynamic models in complex networks with predictior error methods - predictor input selection. IEEE Trans. Automatic Control, april 2016.]

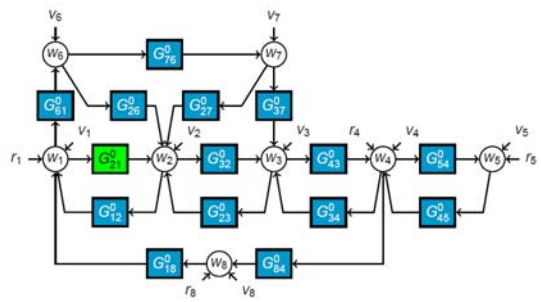
Contents

Towards dynamic network identification

- **Basic identification tools: direct and projection**
  - **From closed-loop to dynamic networks**
- **Single module identification - consistency**
  - **full MISO models**
  - **predictor input selection**
- **Example of decentralized control**
- **Additional results and discussion**

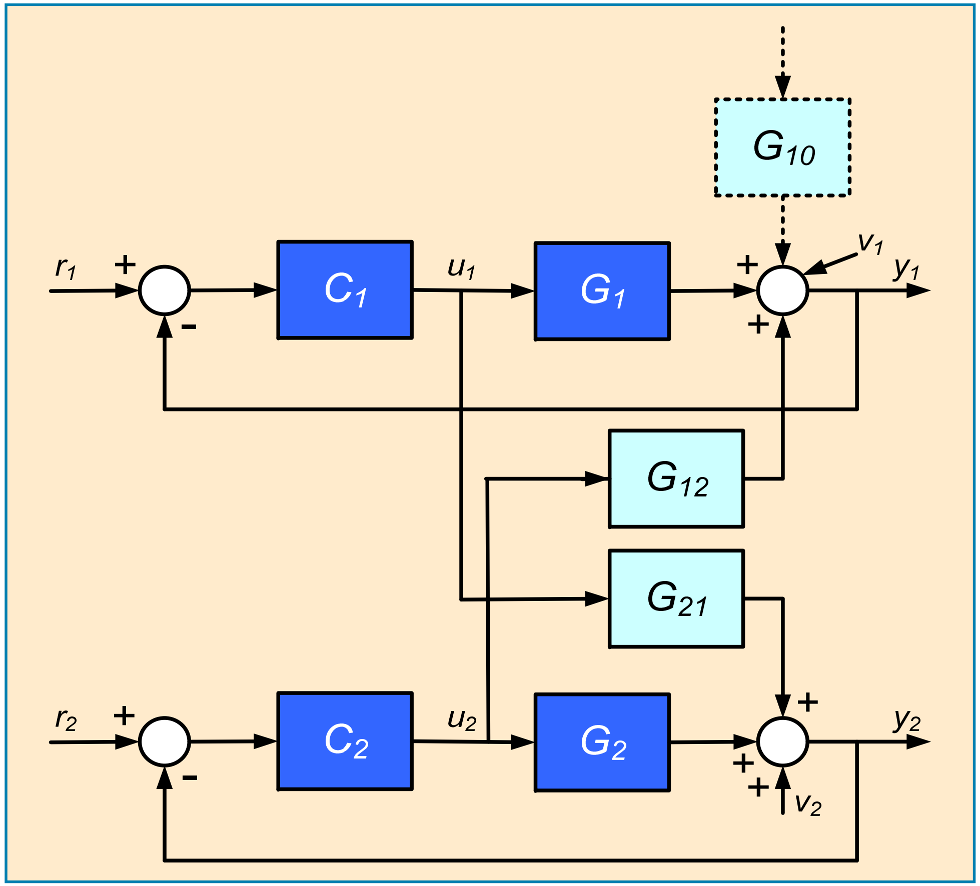






Example Decentralized MPC

Example decentralized MPC; 2 interconnected MPC loops



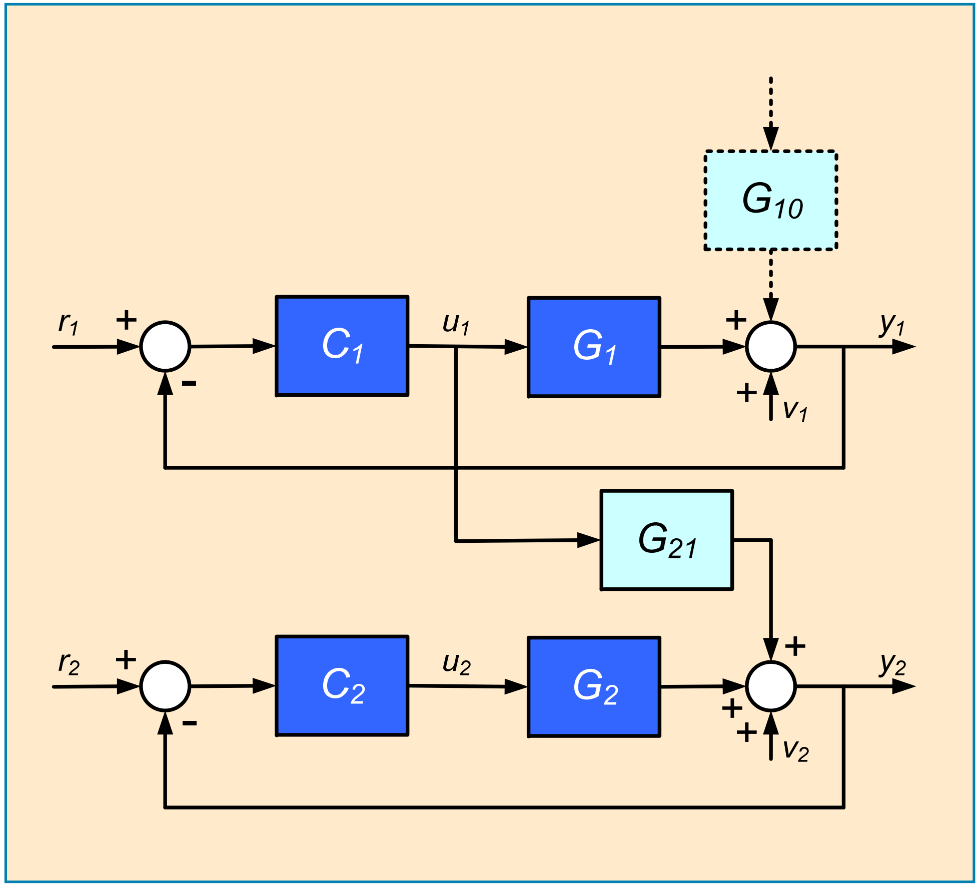
Target:
Identify interaction dynamics
$$G_{21}, G_{12}$$

Addressed by
Gudi & Rawlings (2006)
for the situation $G_{12} = 0$
(no cycles)

Gudi, R. D. and Rawlings, J. B. (2006). Identification for decentralized model predictive control. AIChE Journal, 52(6):2198-2210.

Example decentralized control

Case of Gudi & Rawlings (2006):



Target:
Identify interaction dynamics G_{21}

$$
\begin{aligned}
u_2 &= R_2^i r_2 - R_2^i G_{21} u_1 - R_2^i v_2 \\
y_2 &= S_2^0 G_2 C_2 r_2 + S_2^0 G_{21} u_1 + S_2^0 v_2
\end{aligned}
$$

Options:

1. Identify from $(r_2, u_1) \to u_2$ and find G_{21} by taking the quotient of the two models

2. a) Identify R_2^i from $r_2 \to u_2$

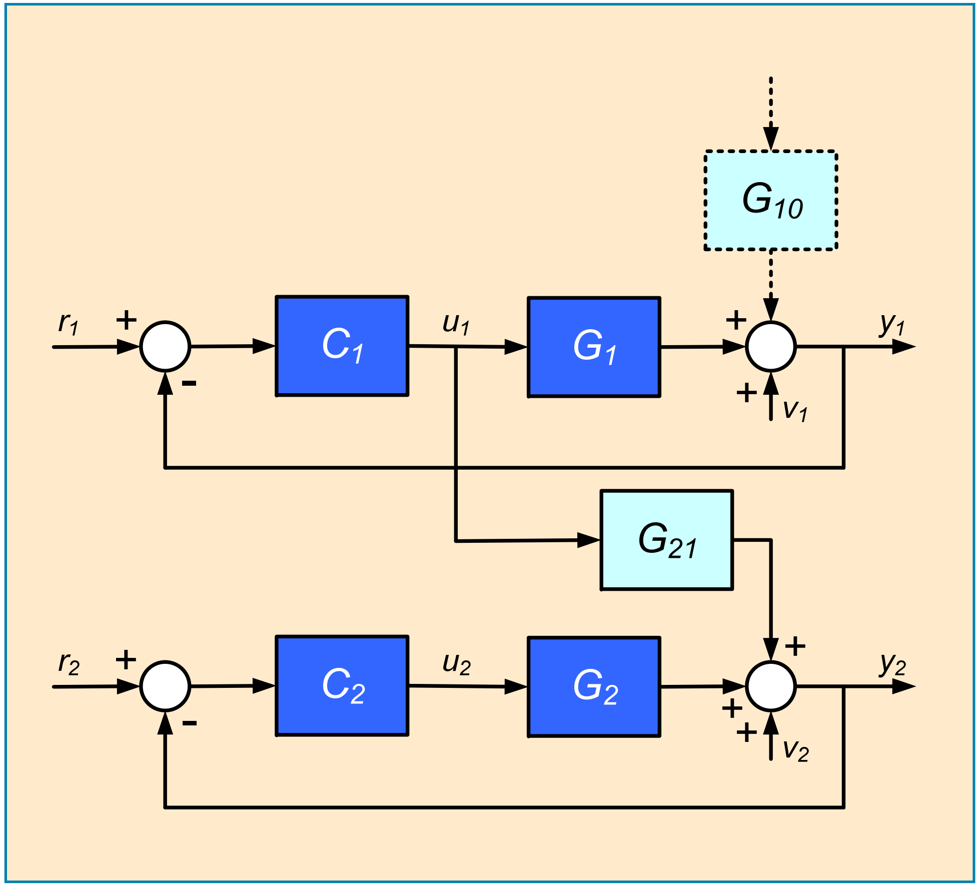
   Simulate: $u_f = (R_2^i)^{-1} u_2$

   b) Identify G_{21} from $u_1 \to u_f$

Excitation through dither signals on r_2 and u_1

Example decentralized control

According to network results (input selection):



$$y_2 = G_{21}u_1 + G_2u_2 + v_2$$

Estimate 2-input 1-output model:

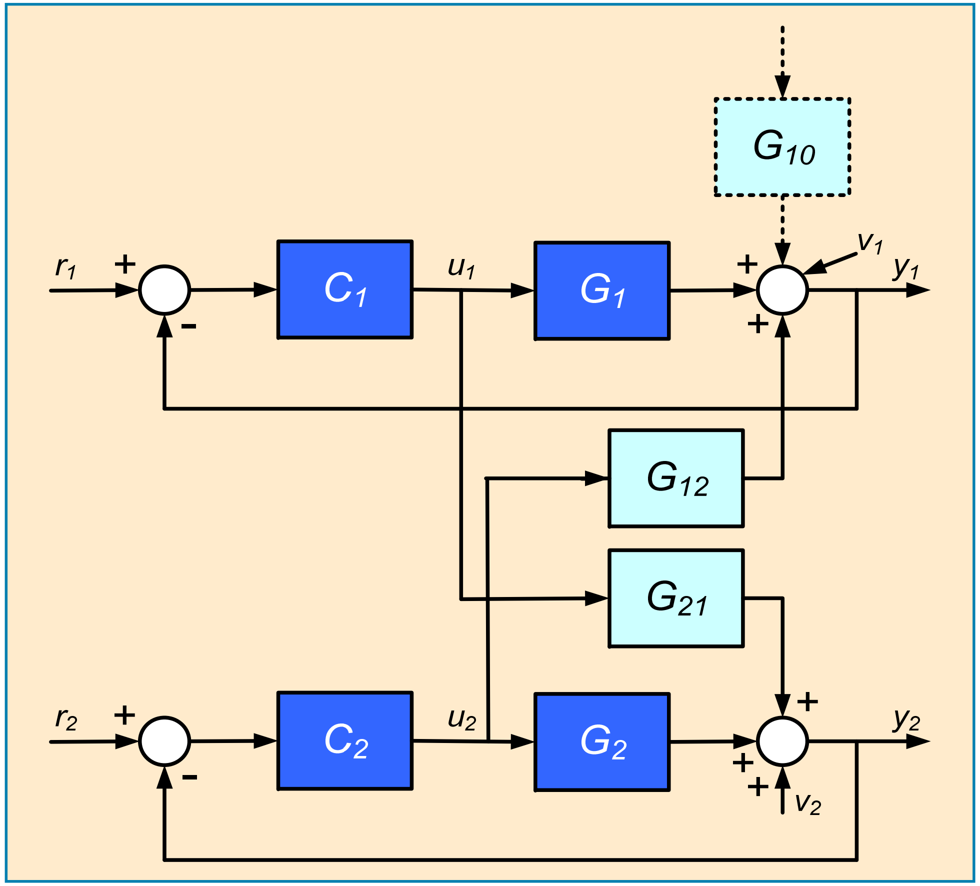
$$(u_1, u_2) \rightarrow y_2$$

provides consistent estimate of G_{21} through both direct and projection method

- Excitation properties of signals remain important:
- Direct method utilizes excitation through noise signals v_1, v_2

Example decentralized control

The more general situation (cyclic connection):



$$y_1 = G_1 u_1 + G_{12} u_2 + v_1$$
$$y_2 = G_{21} u_1 + G_2 u_2 + v_2$$

Estimate 2-input 1-output models:

$$(u_1, u_2) \rightarrow y_1$$
$$(u_1, u_2) \rightarrow y_2$$

provides consistent estimates of

$$G_{21}, G_{12}$$

together with G_1, G_2

If plant models G_1, G_2 are *known* the situation simplifies

Direct method and projection-IV method can handle nonlinear C_i

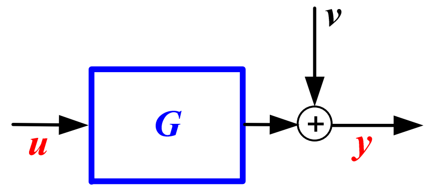
Example decentralized control

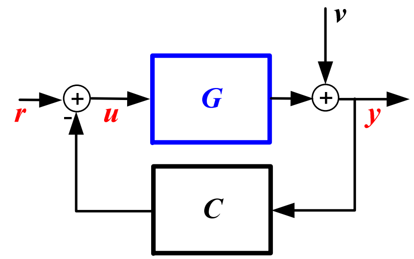
Observation

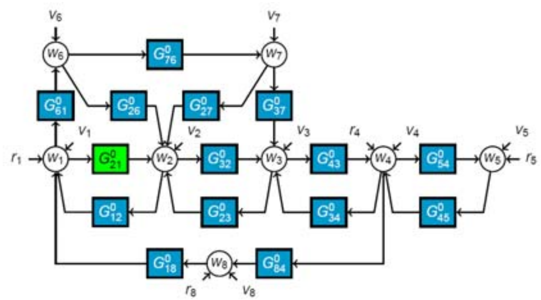
Network identification results provide a formal way to handle these structured identification problems.

Contents

Towards dynamic network identification







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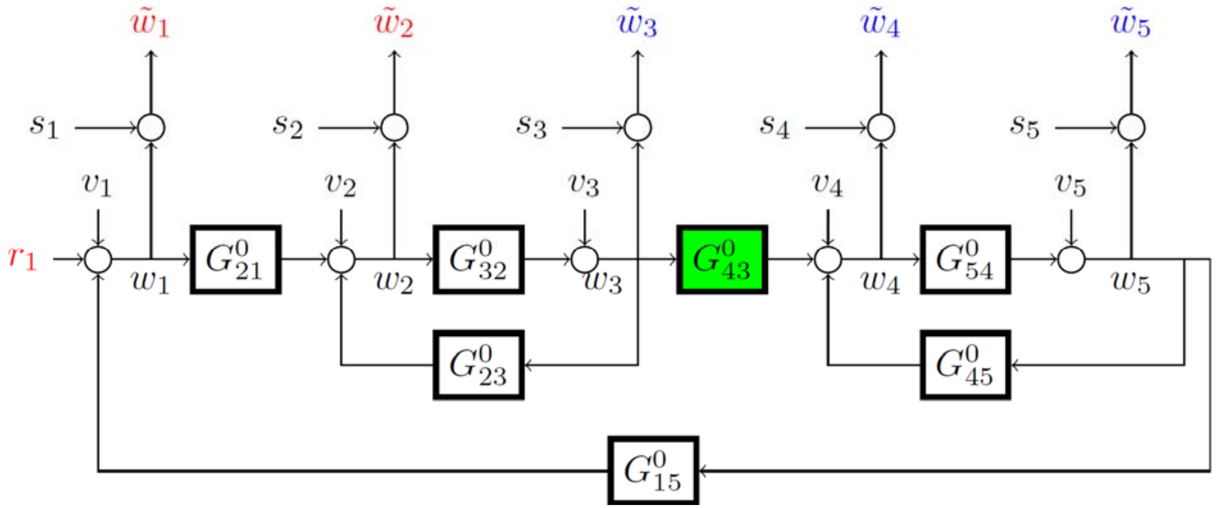
Sensor noise – the errors-in-variables problem

What if node variables are measured with (sensor) noise?



- Classical (tough) problem in open-loop identification
- *More simple* in dynamic networks due to the presence of multiple (correlated) node signals

[A. Dankers et al., Automatica, December 2015]

Network identifiability

Question

Can network models of a full network be distinguished from each other?

Consider: $\boldsymbol{T}(q) = (I - \boldsymbol{G}(q))^{-1} \begin{bmatrix} \boldsymbol{H}(q) & \boldsymbol{R}(q) \end{bmatrix}$

mapping: $\begin{pmatrix} \boldsymbol{e} \\ \boldsymbol{r} \end{pmatrix} \rightarrow \boldsymbol{w}$

For identifiability of a model set, different network models should lead to different $\boldsymbol{T}$ ‘s

This puts conditions on:

- The presence of excitation signals and process noise
- The number of modules that can be parametrized

[H.H.M. Weerts et al, IFAC SYSID 2015, and IFAC ALCOSP 2016]

Discussion / Wrap-up

- So far: focus on (local) consistency results in networks with known structure and linear dynamics

- Many additional questions/topics remain:

  Variance of estimates, influenced by
  - Additional (output) measurements
  - Excitation properties

  [See e.g. work of H. Hjalmarsson, B. Wahlberg, N. Everitt, B. Günes, M. Gevers, A. Bazanella]

- Optimal sensor and actuator locations – experiment design

- Algorithms for application to large-scale systems

Discussion / Wrap-up

- Identification of the structure/topology
  addressed in the literature, in particular forms:
  - Tree-like structures (no loops)
  - Nonparametric methods (Wiener filter)
  - Mostly networks without external excitation and uncorrelated (white) process noises on every node

  see e.g. Materassi, Innocenti (TAC-2010), Chiuso and Pillonetto (Automatica, 2012)

- Sparse identification methods can be used in an identification setting to identify the topology (non-zero transfers)

- New identifiability concepts apply to the unique determination of a network topology
  see e.g. Goncalves & Warnick (TAC-2008), Weerts et al. (SYSID-2015).

- Connection with decentralized/distributed control

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Co-workers:

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Bibliography

- P.M.J. Van den Hof, A. Dankers, P. Heuberger and X. Bombois (2013). Identification of dynamic models in complex networks with prediction error methods - basic methods for consistent module estimates. *Automatica,* Vol. 49, no. 10, pp. 2994-3006.
- A. Dankers, P.M.J. Van den Hof, X. Bombois and P.S.C. Heuberger (2015). Errors-in-variables identification in dynamic networks - consistency results for an instrumental variable approach. *Automatica*, Vol. 62, pp. 39-50.
- B. Günes, A. Dankers and P.M.J. Van den Hof (2014). Variance reduction for identification in dynamic networks. Proc. 19th IFAC World Congress, 24-29 August 2014, Cape Town, South Africa, pp. 2842-2847.
- A.G. Dankers, P.M.J. Van den Hof, P.S.C. Heuberger and X. Bombois (2012). Dynamic network structure identification with prediction error methods - basic examples. Proc. 16th IFAC Symposium on System Identification (SYSID 2012), 11-13 July 2012, Brussels, Belgium, pp. 876-881.
- A.G. Dankers, P.M.J. Van den Hof and X. Bombois (2014). An instrumental variable method for continuous-time identification in dynamic networks. Proc. 53rd IEEE Conf. Decision and Control, Los Angeles, CA, 15-17 December 2014, pp. 3334-3339.
- H.H.M. Weerts, A.G. Dankers and P.M.J. Van den Hof (2015). Identifiability in dynamic network identification. Proc.17th IFAC Symp. System Identification, 19-21 October 2015, Beijing, P.R. China.
- A. Dankers, P.M.J. Van den Hof, P.S.C. Heuberger and X. Bombois (2016). Identification of dynamic models in complex networks with predictior error methods - predictor input selection. *IEEE Trans. Automatic Control*, *61 (4)*, pp. 937-952, April 2016.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2016). Identifiability of dynamic networks with part of the nodes noise-free. Proc. 12th IFAC Intern. Workshop ALCOSP 2016, June 29 - July 1, 2016, Eindhoven, The Netherlands.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2016). Identifiability of dynamic networks with noisy and noise-free nodes. ArXiv:1609.00864 [CS.sy]
- P.M.J. Van den Hof, H.H.M. Weerts and A.G. Dankers (2016). Prediction error identification with rank-reduced output noise. Submitted to 2017 American Control Conference, 24-26 May 2017, Seattle, WA, USA.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2016). Identification of dynamic networks with rank-reduced process noise. Submitted to 2017 IFAC World Congress, 9-14 July 2017, Toulouse, France.

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