COLLECTIVE MOTION:

T. Vicsek

http://hal.elte.hu/~vicsek

Collective behaviour-> Collective decision-making-> Collective motion

≈ 60% based on: Physics Reports 517 (2012) 71–140 **with A.Zafeiris**

So, for references see the above text or my home page

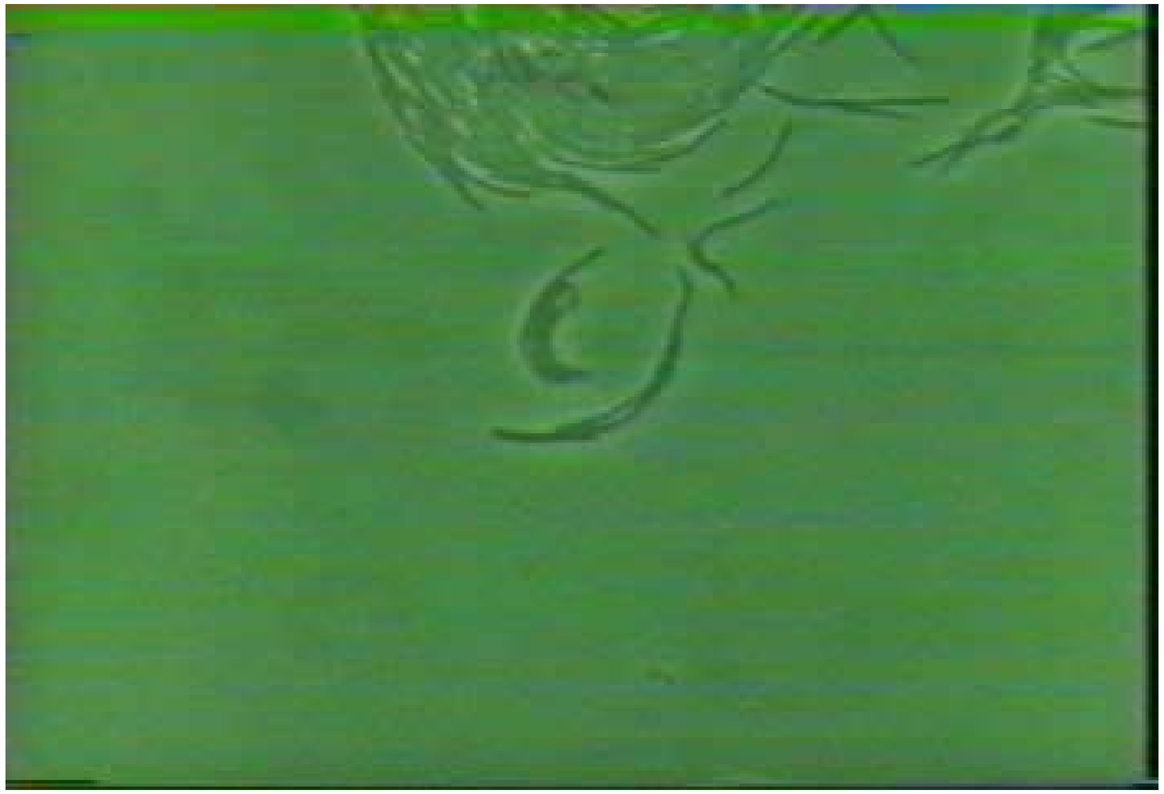
(would be too much to include here)

PART I

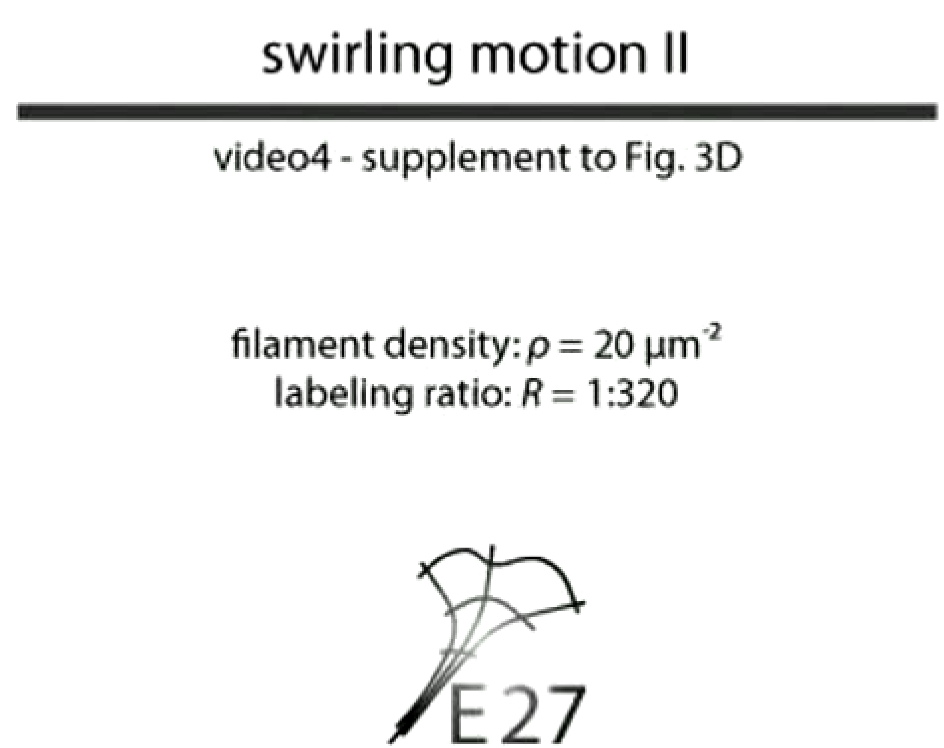
Introduction to collective motion:

Observations, basic experiments and the simplest models

Collective motion of



From BBC (I. Couzin)

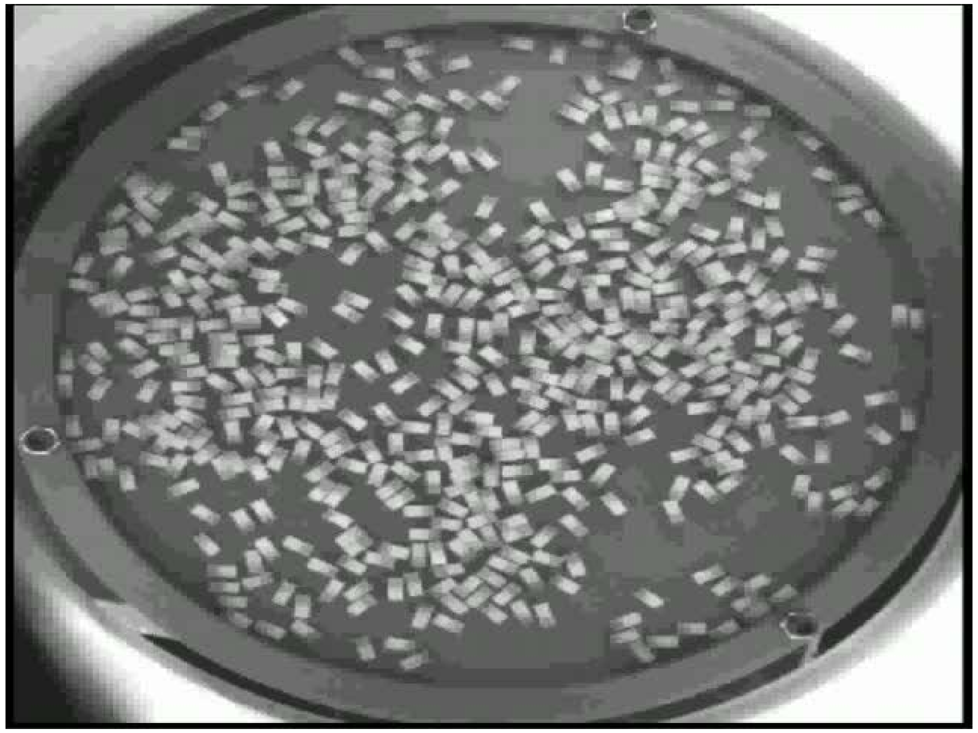


Bausch group, Munich

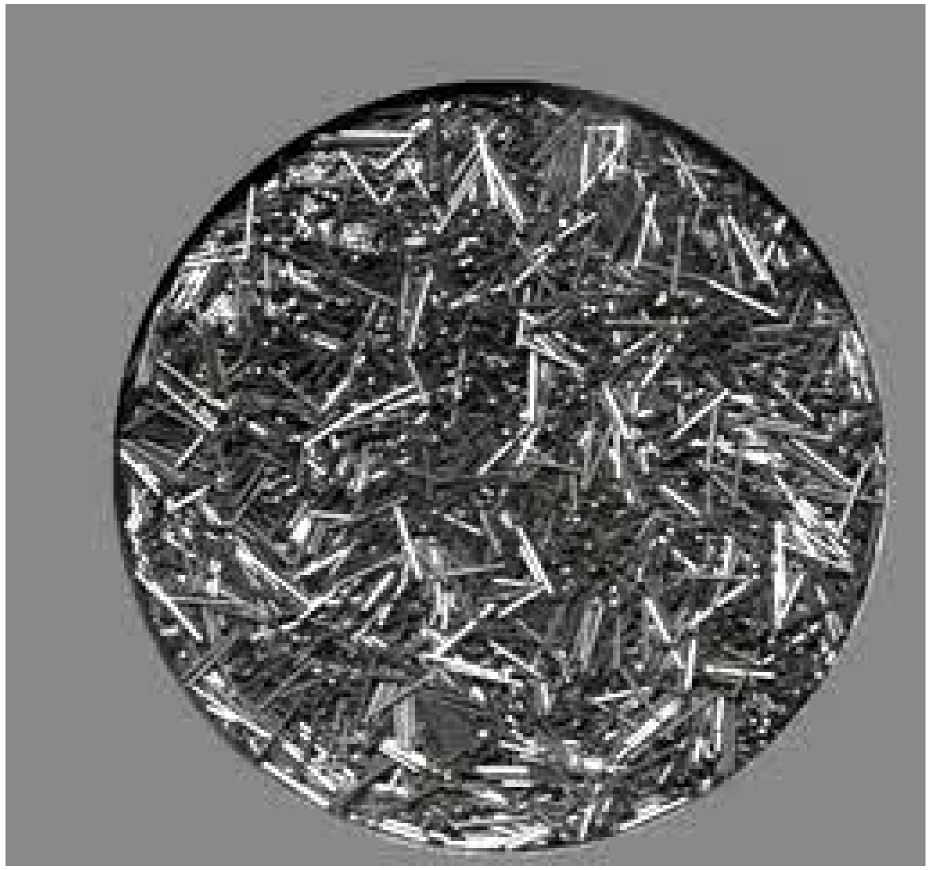



Collective motion of rods (physics)
shaken from below

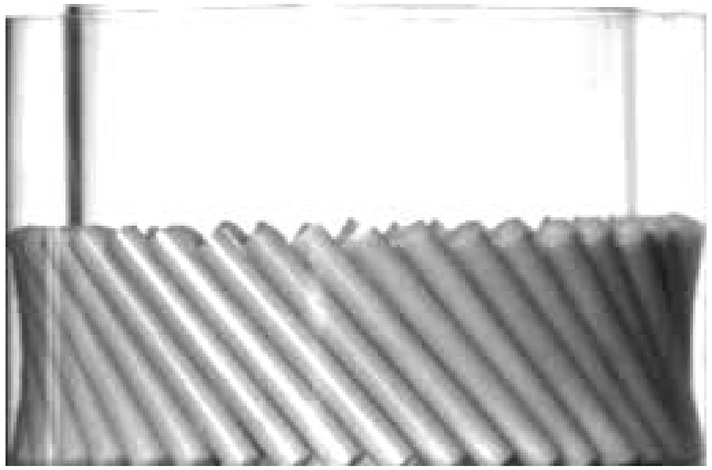
Asymmetric units



Just simple rods



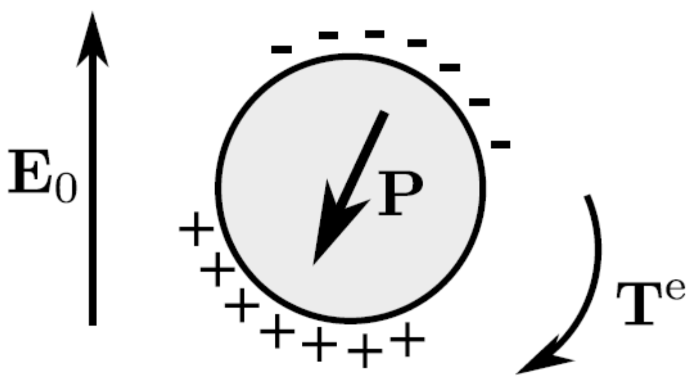
Kudrolli et al PRL,
(2003), (2008)

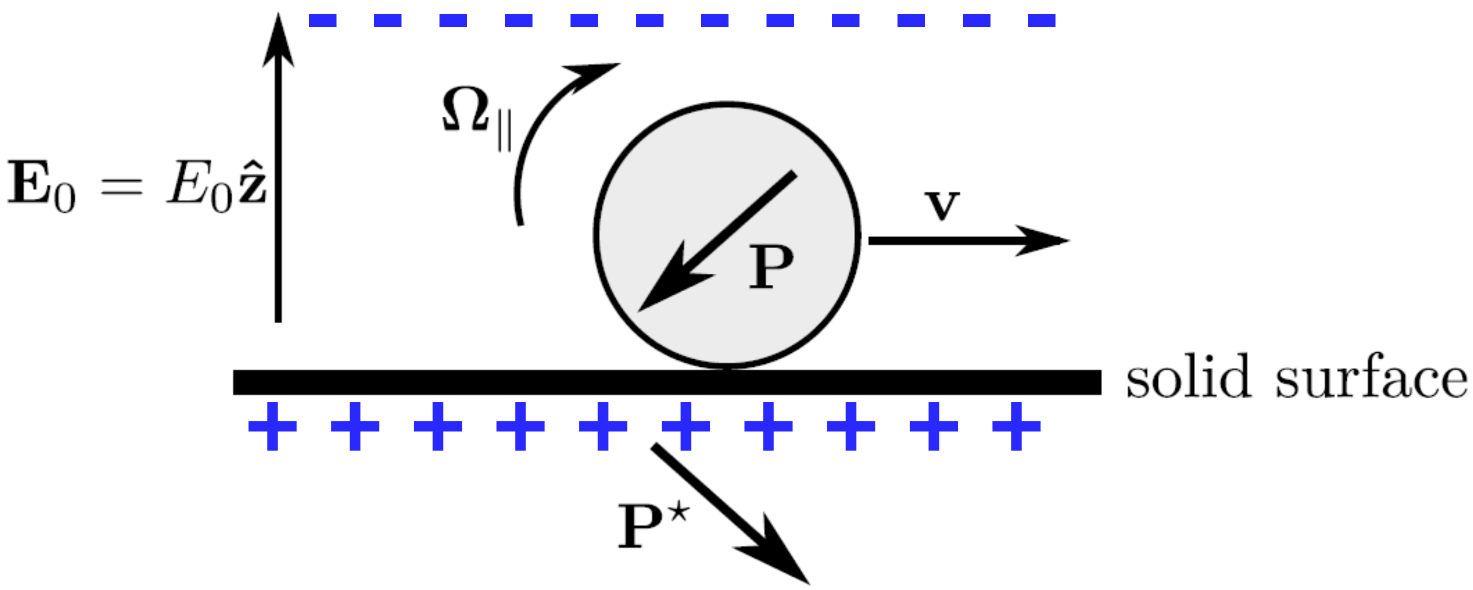


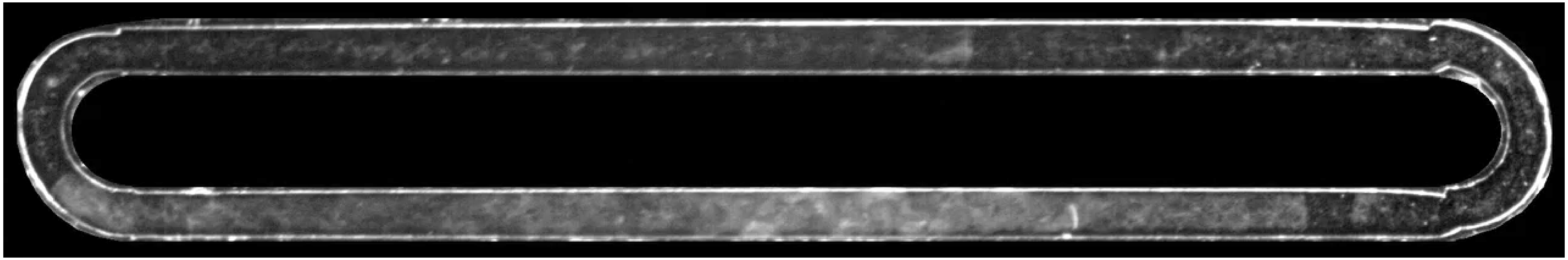
A recent example from physics

Quincke rotation on a planar surface
in liquid dieletrics

Bicard et al Nature 2013





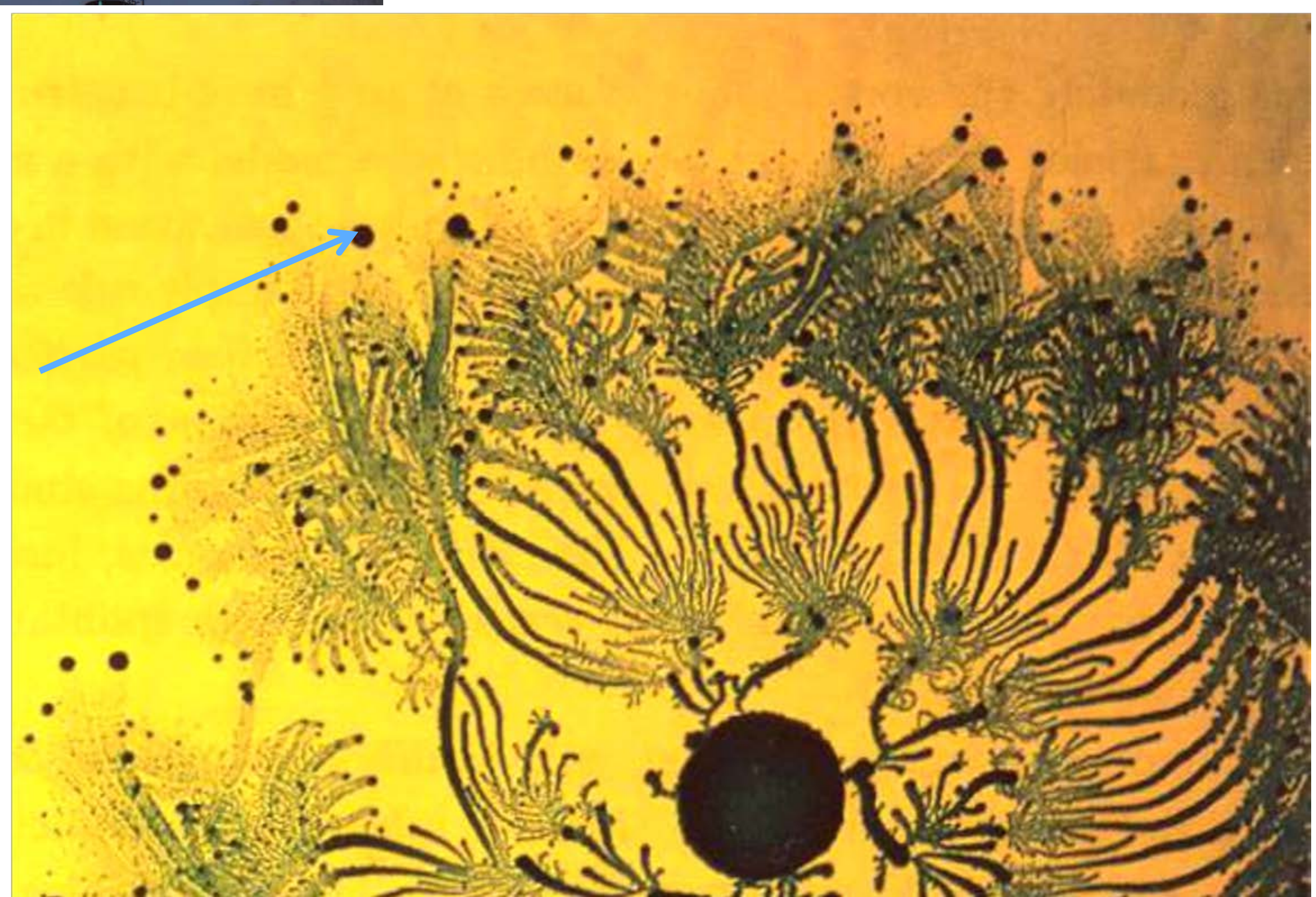


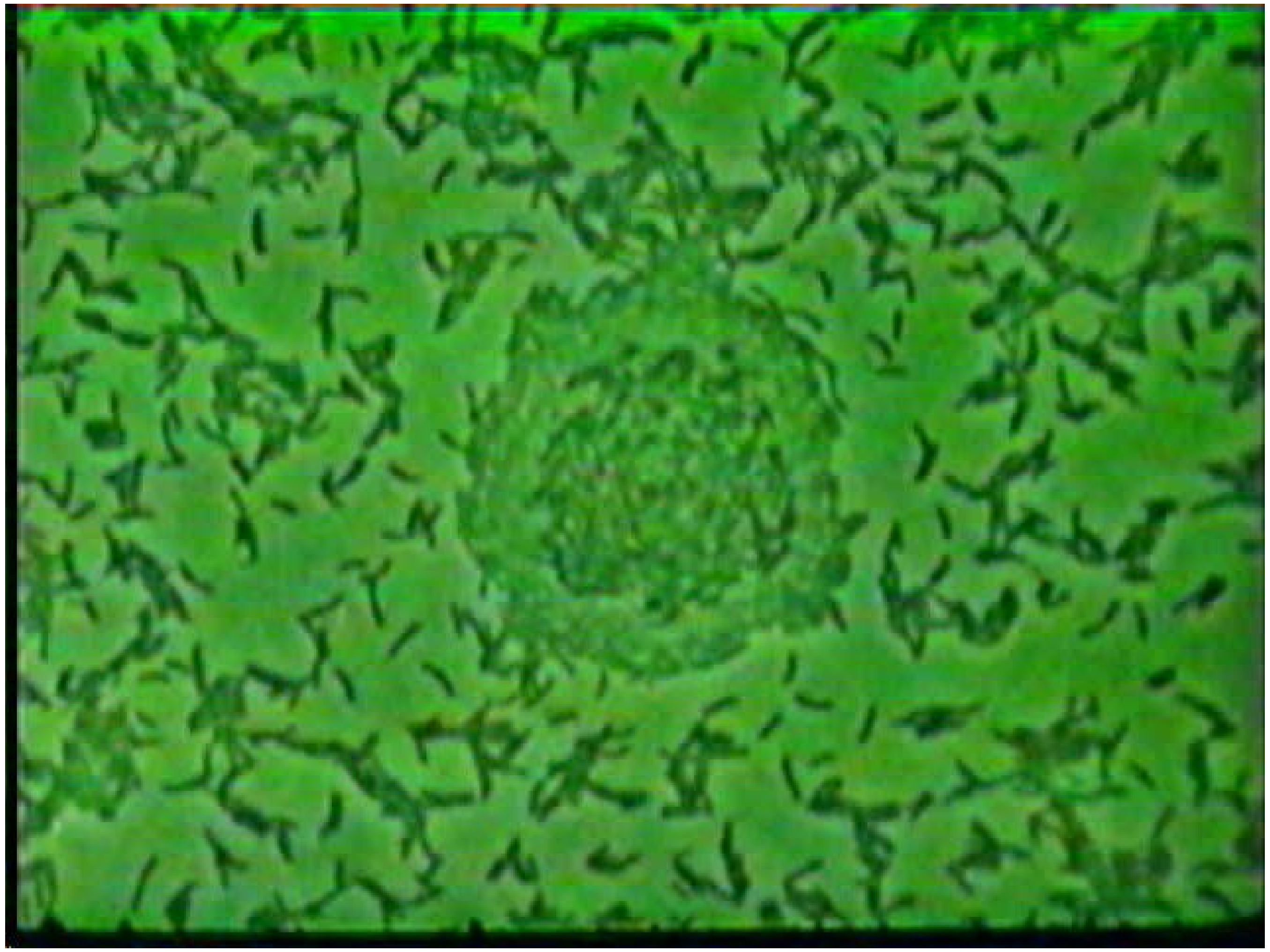
Millions of little (1micrometer) spheres

Mallards (a kind of duck) winter at Balaton

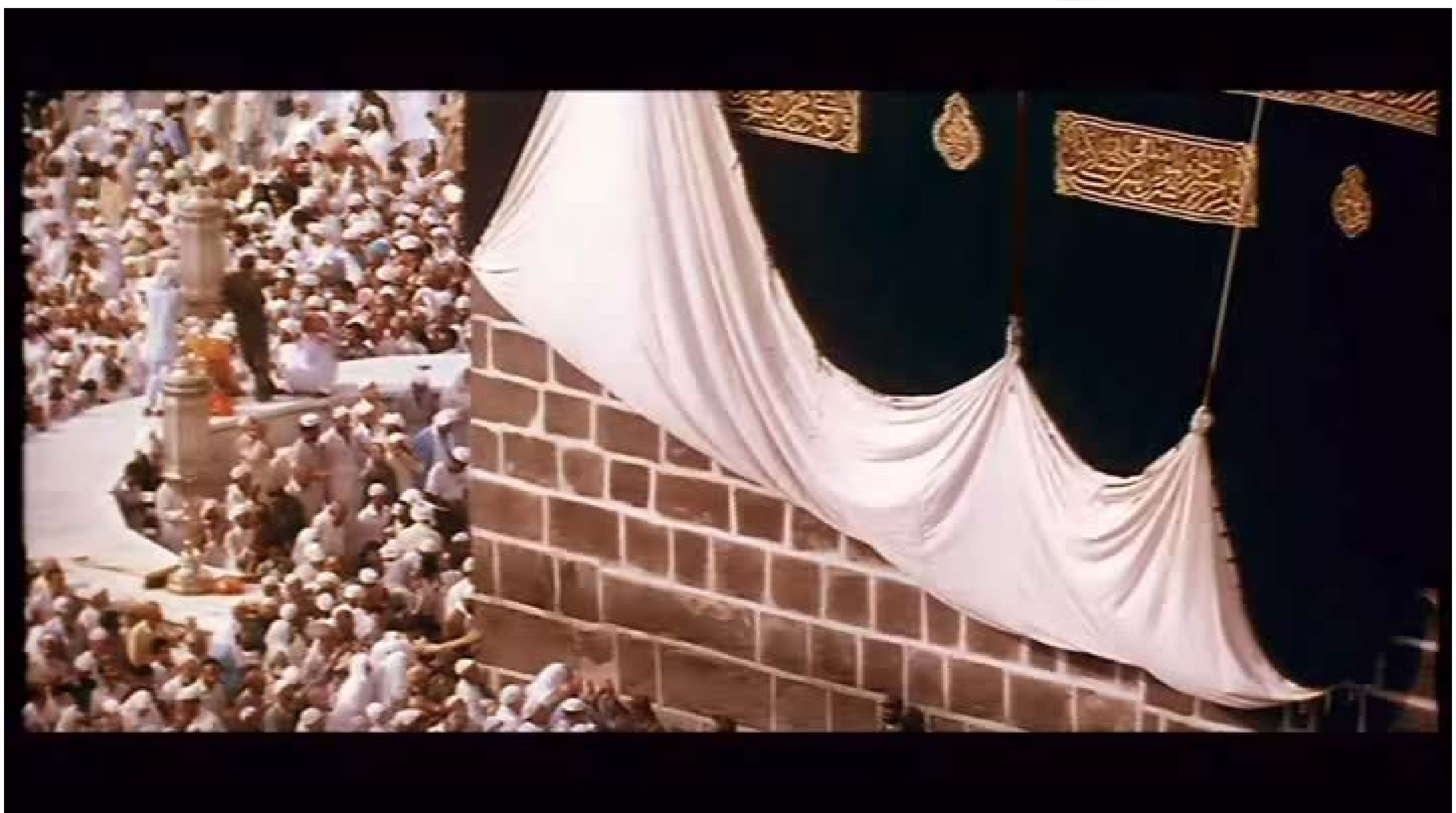
A universal pattern of motion

Locusts (Buhl, Sumpter, Couzin et al, *Science*, 2006)






Self-Organized Flocking
of Kobots in a Closed Arena
Ali E. Turgut, Hande Çelikkanat,
Fatih Gökçe and Erol Şahin
KOVAN Research Lab.
Dept. of Computer Eng.
Middle East Technical University
Ankara, Turkey



Observation: complex units exhibit simple collective behaviours (collective motion patterns)

and

simple units produce complex patterns

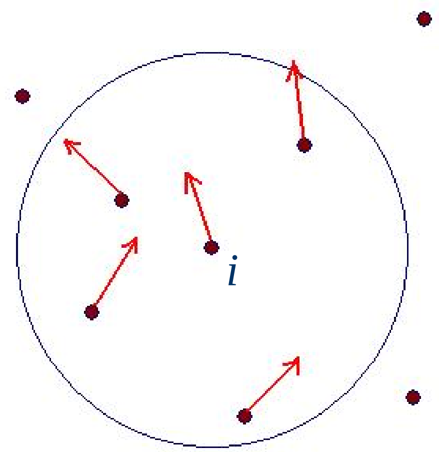
Our goals are:
- classification of patterns
- finding the basic laws (microscopic versus global)

} of collective motion

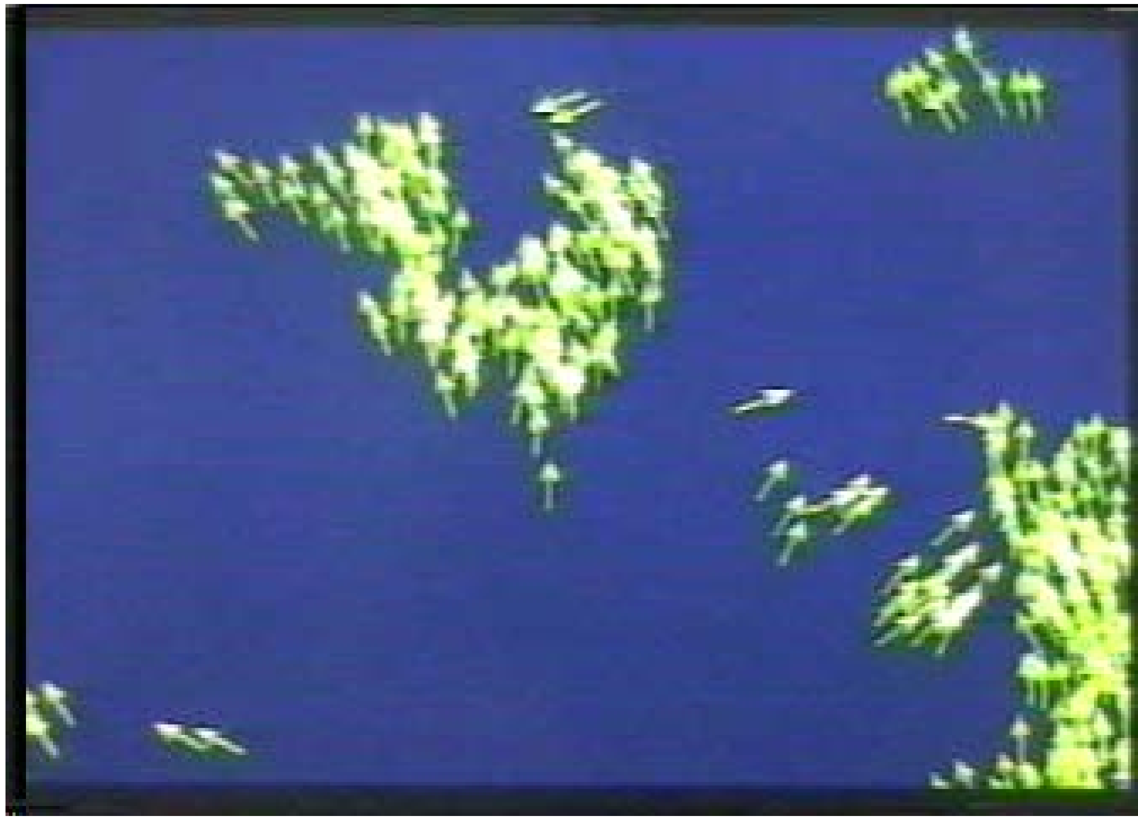
„Universality" (versus specificity)

Swarms, flocks and herds

- **Model* (SPP) :** The particles

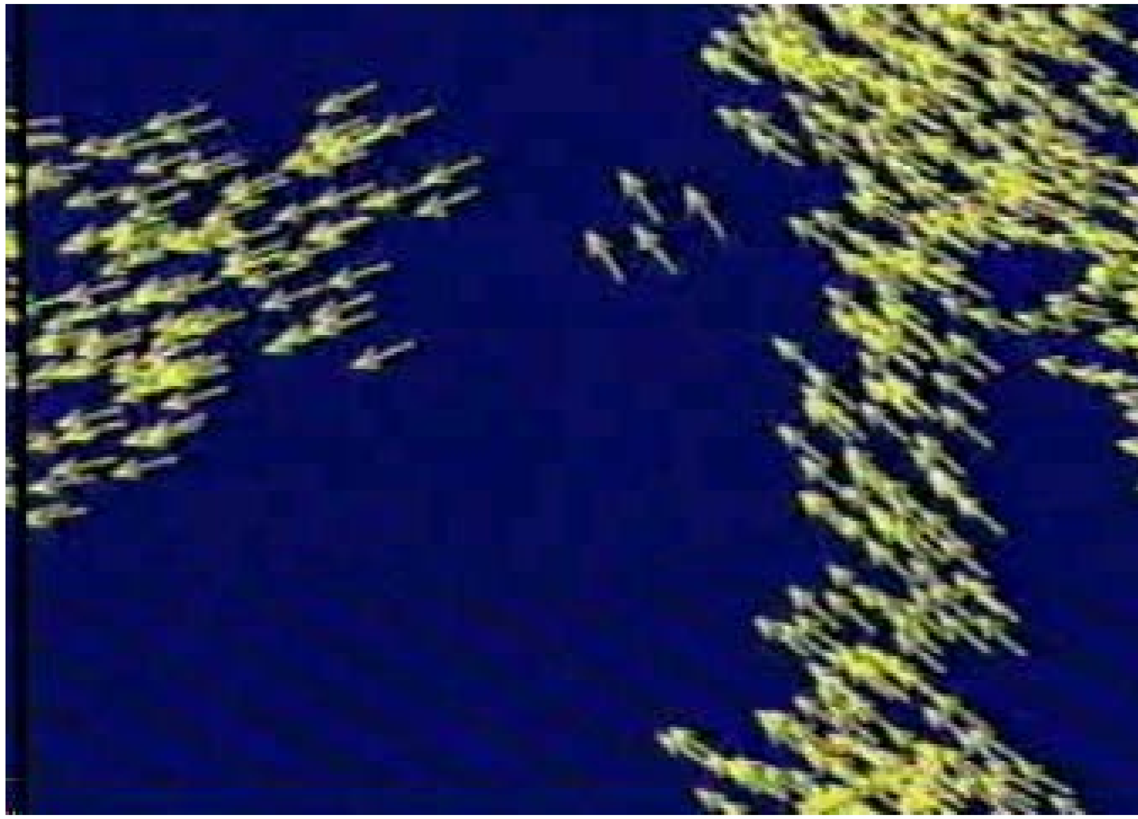


- maintain a given absolute value of the velocity v_0
- follow their neighbours the „**ALIGNMENT RULE**"
- motion is perturbed by fluctuations □



$$\vec{e}_i\,(t+1) = E\left[E[\langle \vec{e}_j\,(t)\rangle_j\,] + \vec{\eta}(t)\right]$$

(*E* normalizes the magnitude into unity)



- Follow the neighbours rule is an abstract way to take into account interactions of very different possible origins

- **Result: *ordering is due to motion***

* T.V, A. Czirok, E. Ben-Jacob and I. Cohen, PRL, 1995

$$\vec{v}_i(t+1) = v_0 \frac{\langle \vec{v}_j(t) \rangle_R}{\left| \langle \vec{v}_j(t) \rangle_R \right|} + perturbation$$

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1).$$

The rule for the direction is equivalent to calculating the angle ϑ_i corresponding to the direction of motion $\vec{v}_i$ from

$$\vartheta_i(t) = \arctan\left[\frac{\langle v_{j,x} \rangle_R}{\langle v_{j,y} \rangle_R}\right], \text{as}$$

$$\vartheta_i(t+1) = \vartheta_i(t) + \Delta_i(t),$$

Just trying to keep going with v_0 *and* repelling force (*no alignment rule*)

$$\frac{d\vec{v}_i}{dt} = \vec{v}_i \left(\frac{v_0}{|\vec{v}_i|} - 1 \right) + \vec{F}_i + \vec{\xi}_i$$

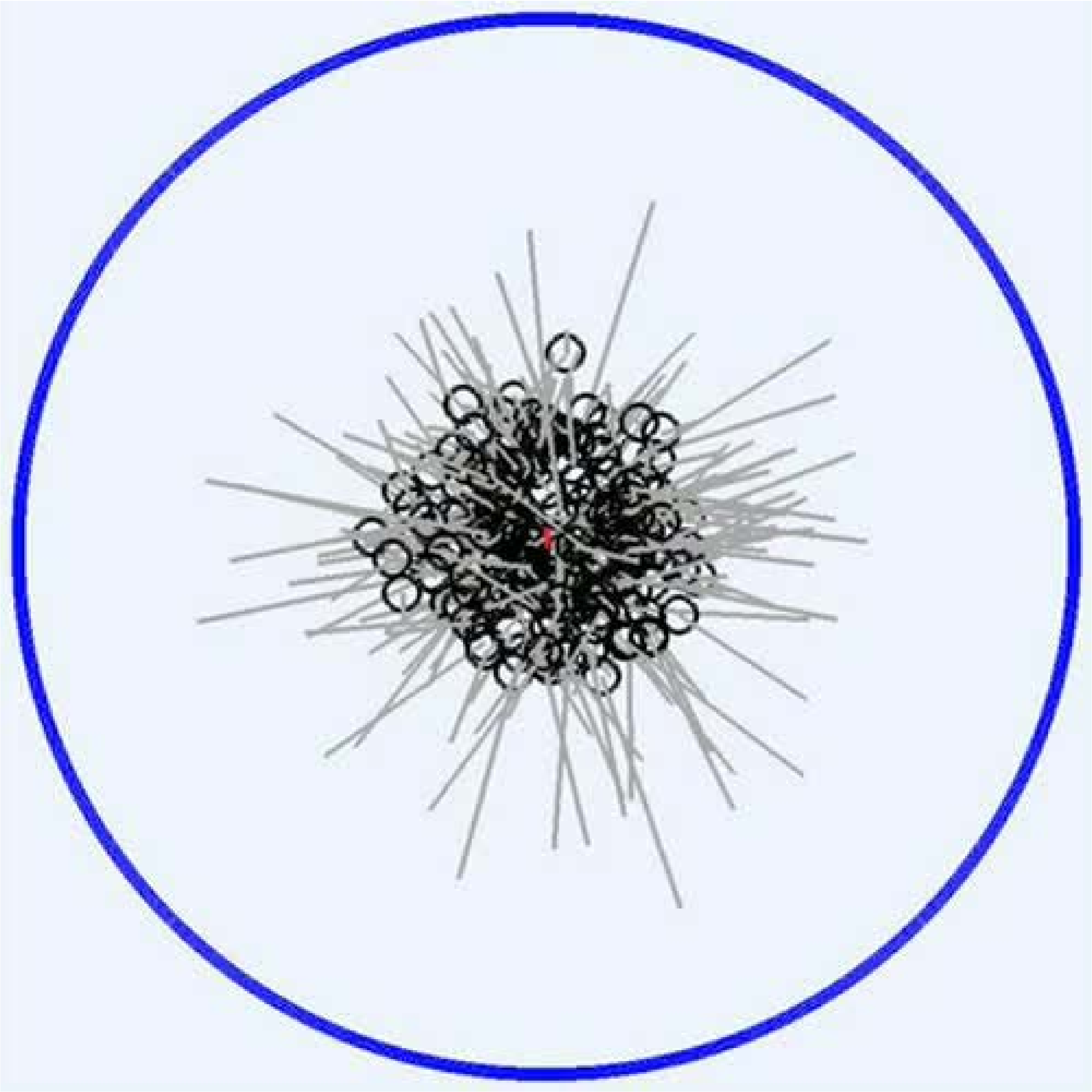
$\vec{\xi}_i$ White noise

$$\vec{F}_i = \sum_{i \neq j} \vec{F}_{ij} + \vec{F}_i\,(\text{wall})$$

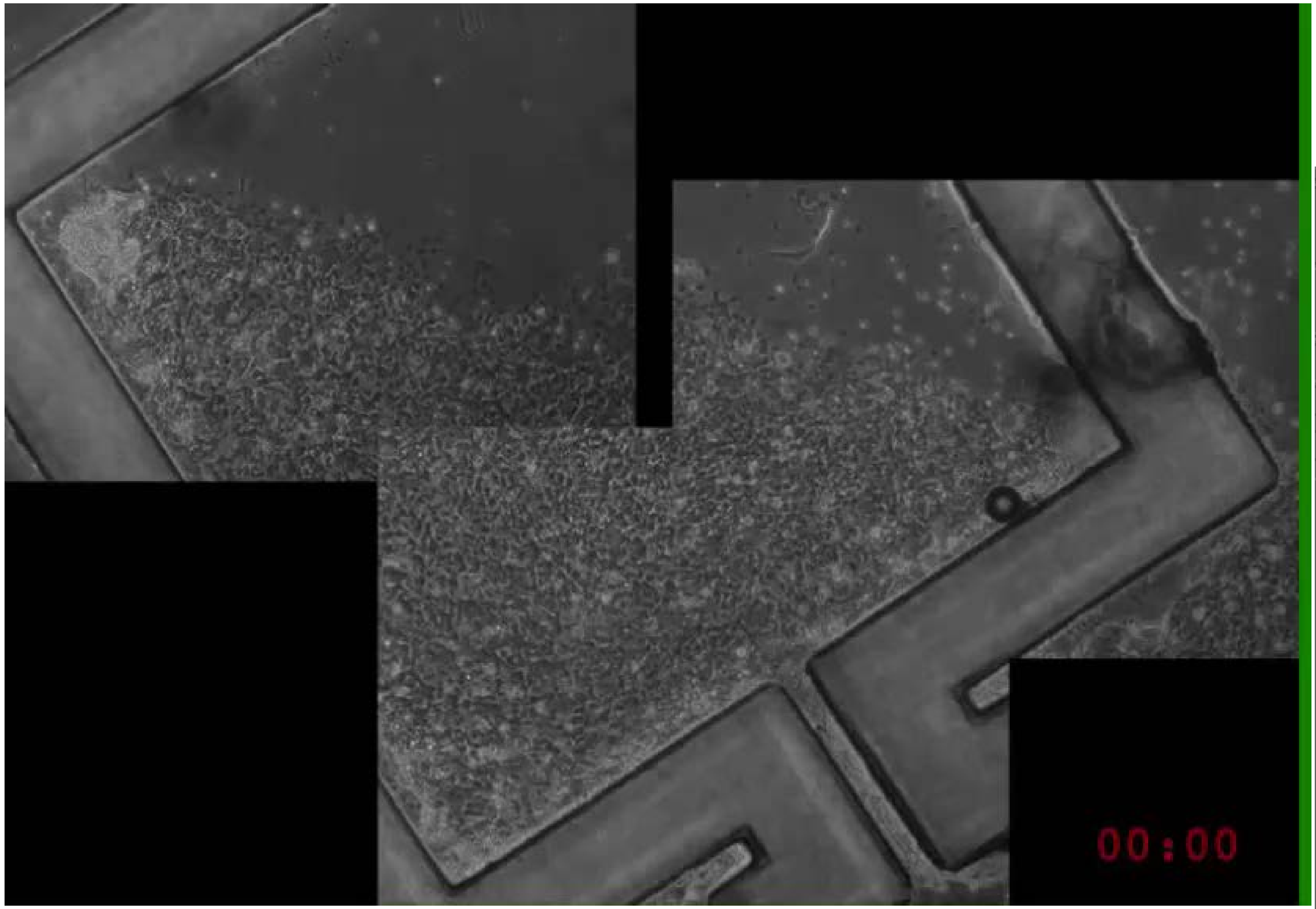
$$\vec{r}_{ij} = \vec{x}_i - \vec{x}_j$$

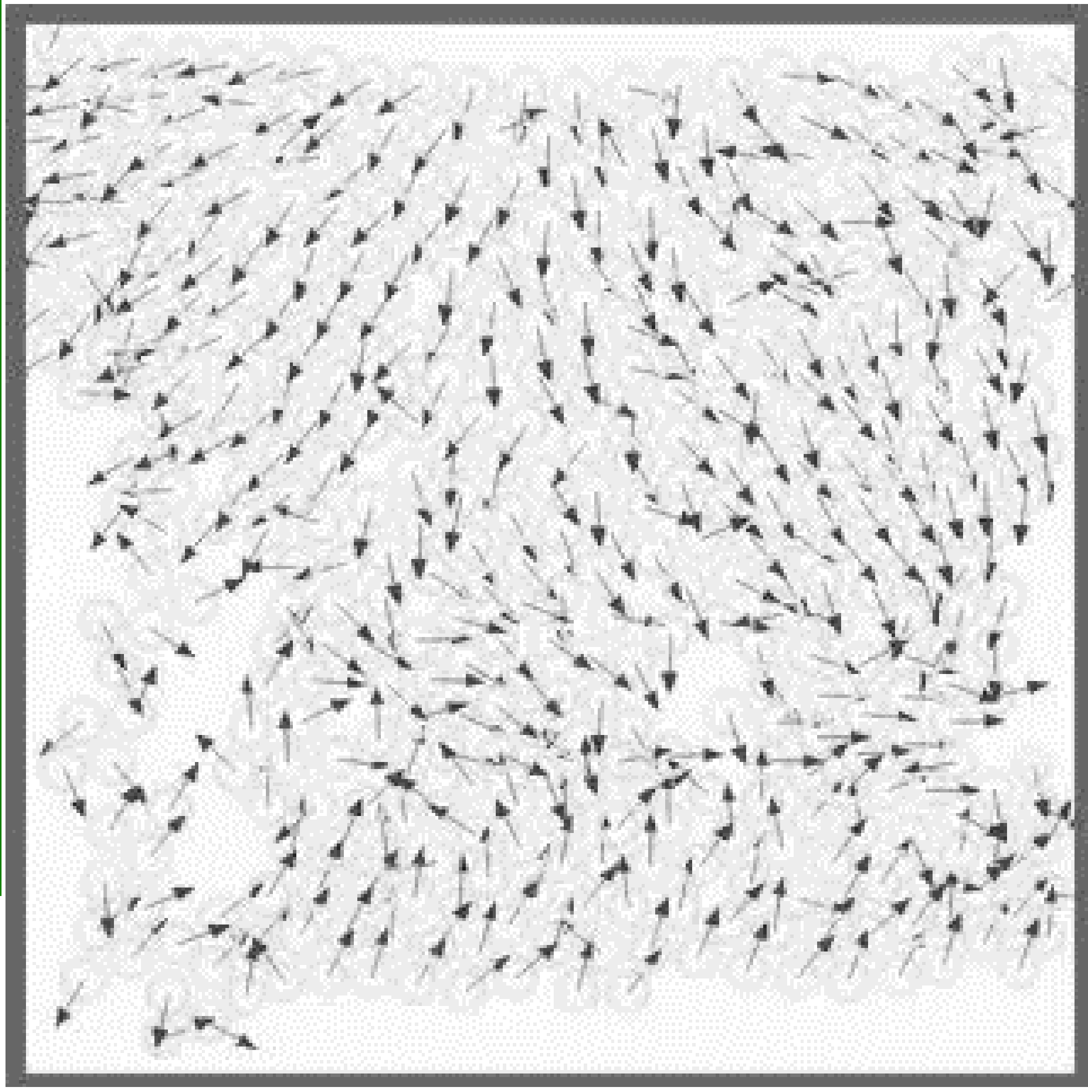
$$\vec{F}_{ij} = \begin{cases} C\vec{r}_{ij} \left(\frac{r_0}{|\vec{r}_{ij}|} - 1 \right), & \text{if } |\vec{r}_{ij}| \leq r_0, \text{ and} \\ 0, & \text{otherwise.} \end{cases}$$

„sudden” ordering



But, e.g., for cells:





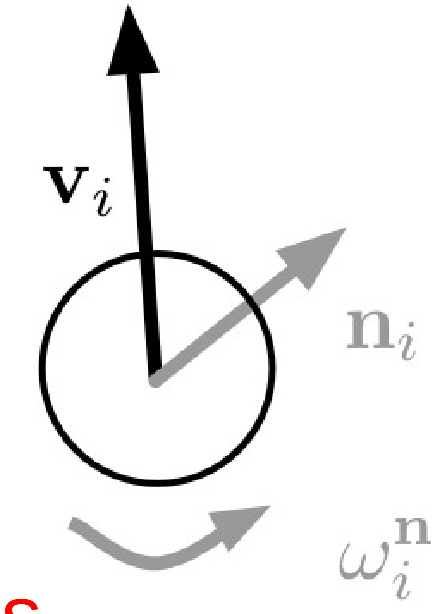
We simulate this

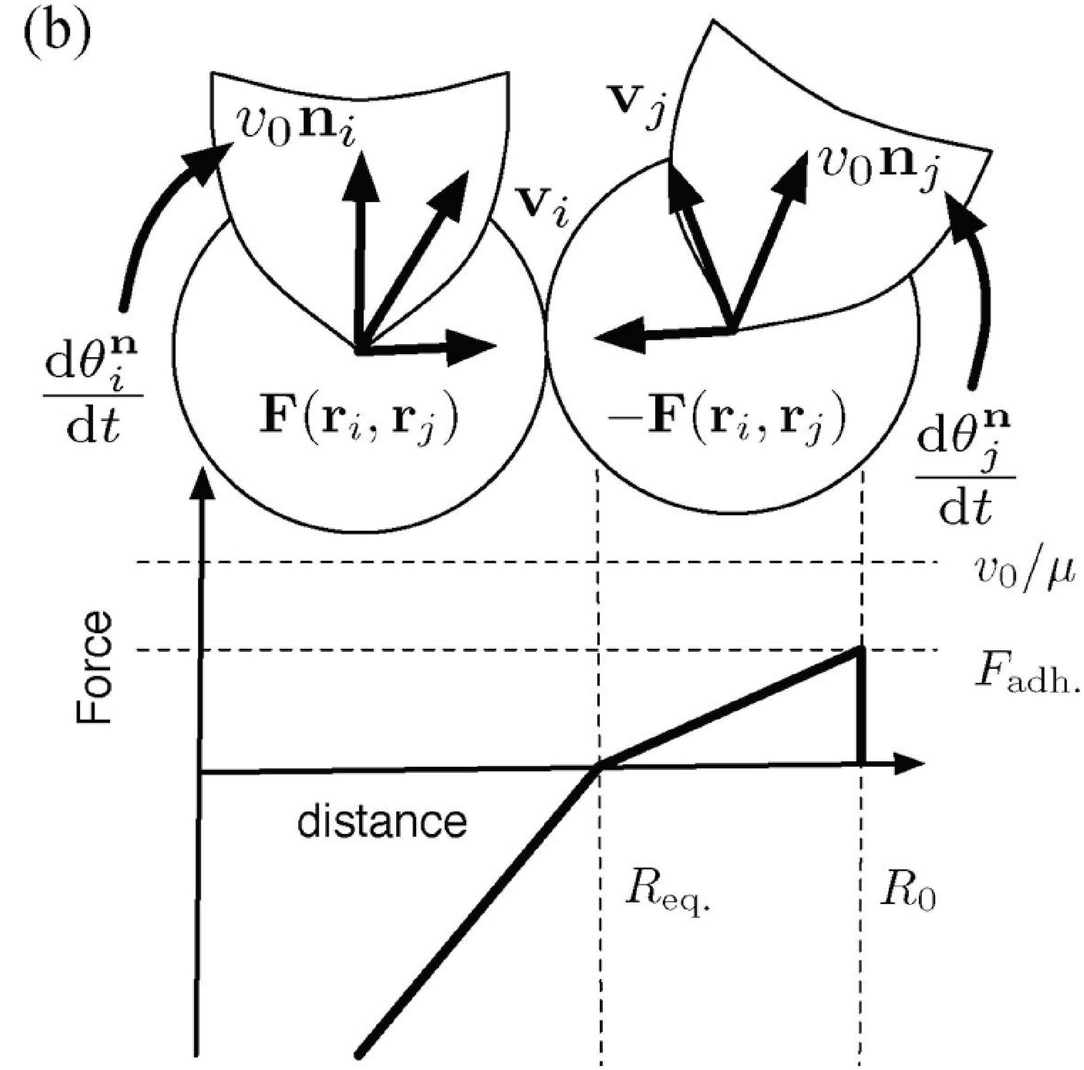
by assuming soft pushing and adherence

resulting in a

relaxation (with a characteristic time tau) of the „preferred” (when left alone) direction to the actual one

Qualitatively new feature:
the velocity of the neighbours
is not part of the equations





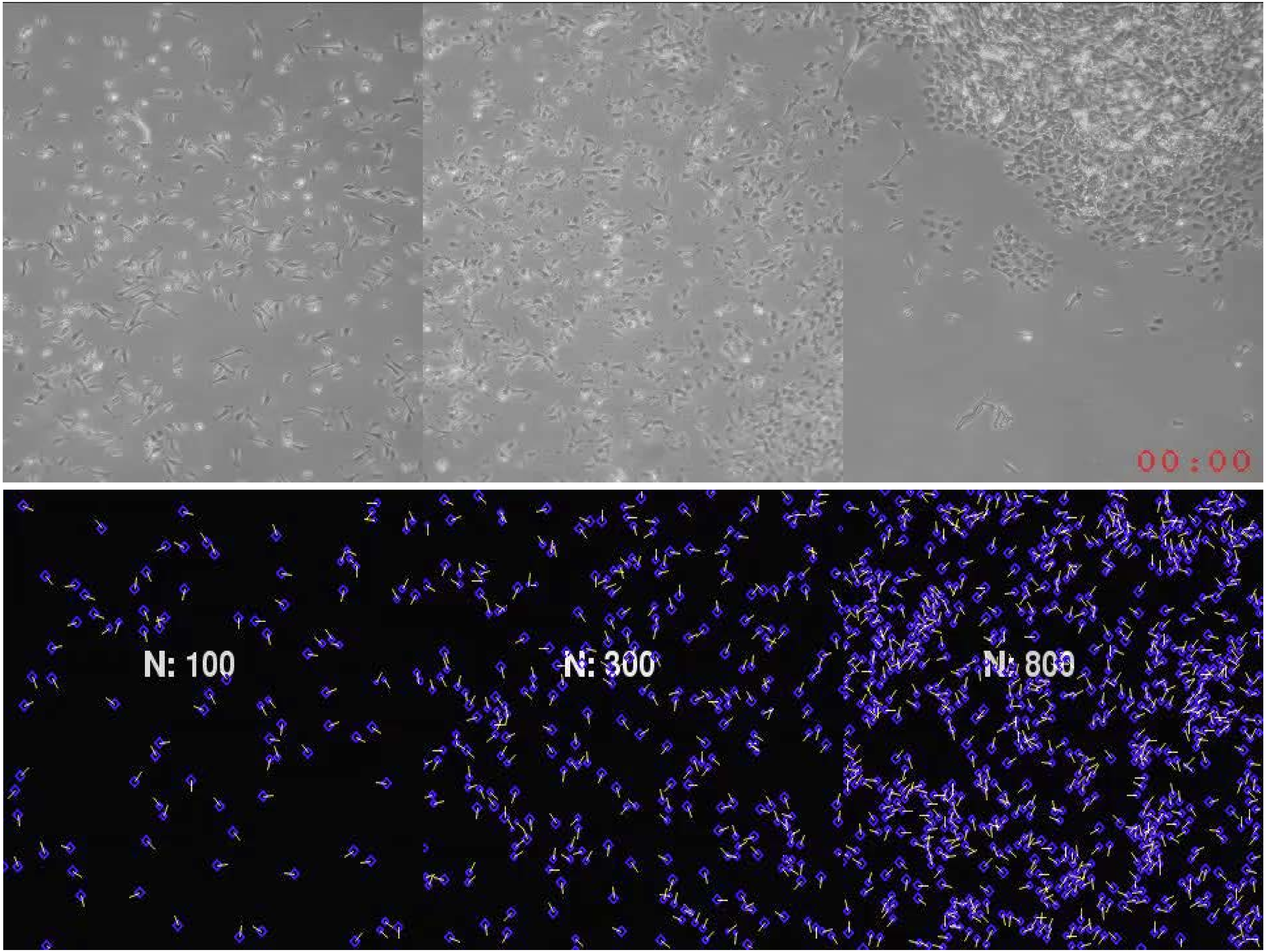
$$\boldsymbol{v}_i = \frac{d\mathbf{r}_i}{dt} = \sum_j \mathbf{F}_{ij} + v_0\mathbf{n}_i$$

$$\omega_i = \frac{d\vartheta_i^{\mathbf{n}}}{dt} = \frac{1}{\tau}\sin^{-1}\left(\left\|\mathbf{n}_i \times \frac{\mathbf{v}_i}{v_i}\right\|\right) + \xi$$

$$\xi \in [\eta/2, \eta/2]$$

$\mathbf{v}_i$ actual velocity of cell i
$\mathbf{F}_{ij}$ interaction force (repulsion/adhesion)
$\mathbf{n}_i$ preferred direction of motion
ω_i rate of change of $\mathbf{n}_i$

The preferred direction of motion of a cell is approaching the actual direction with a rate τ


00:00
N: 100
N: 300
N: 800

Lessons:

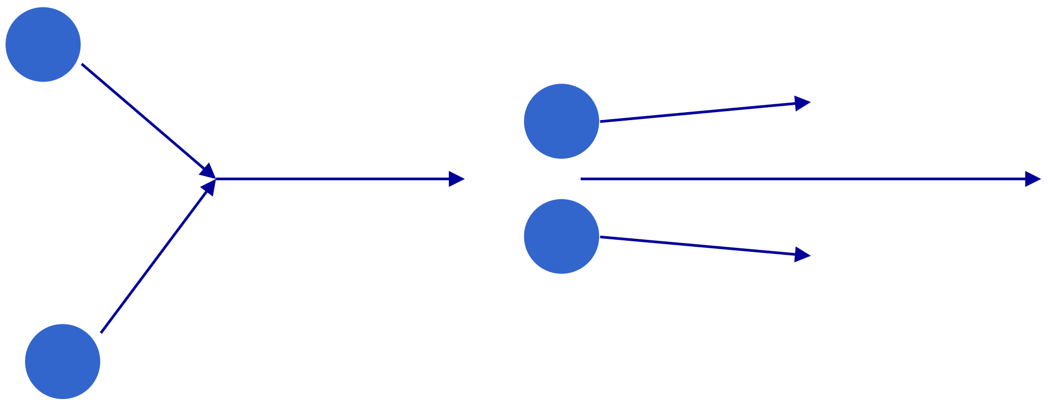
1. Most **patterns** of collective motion are ***universal***

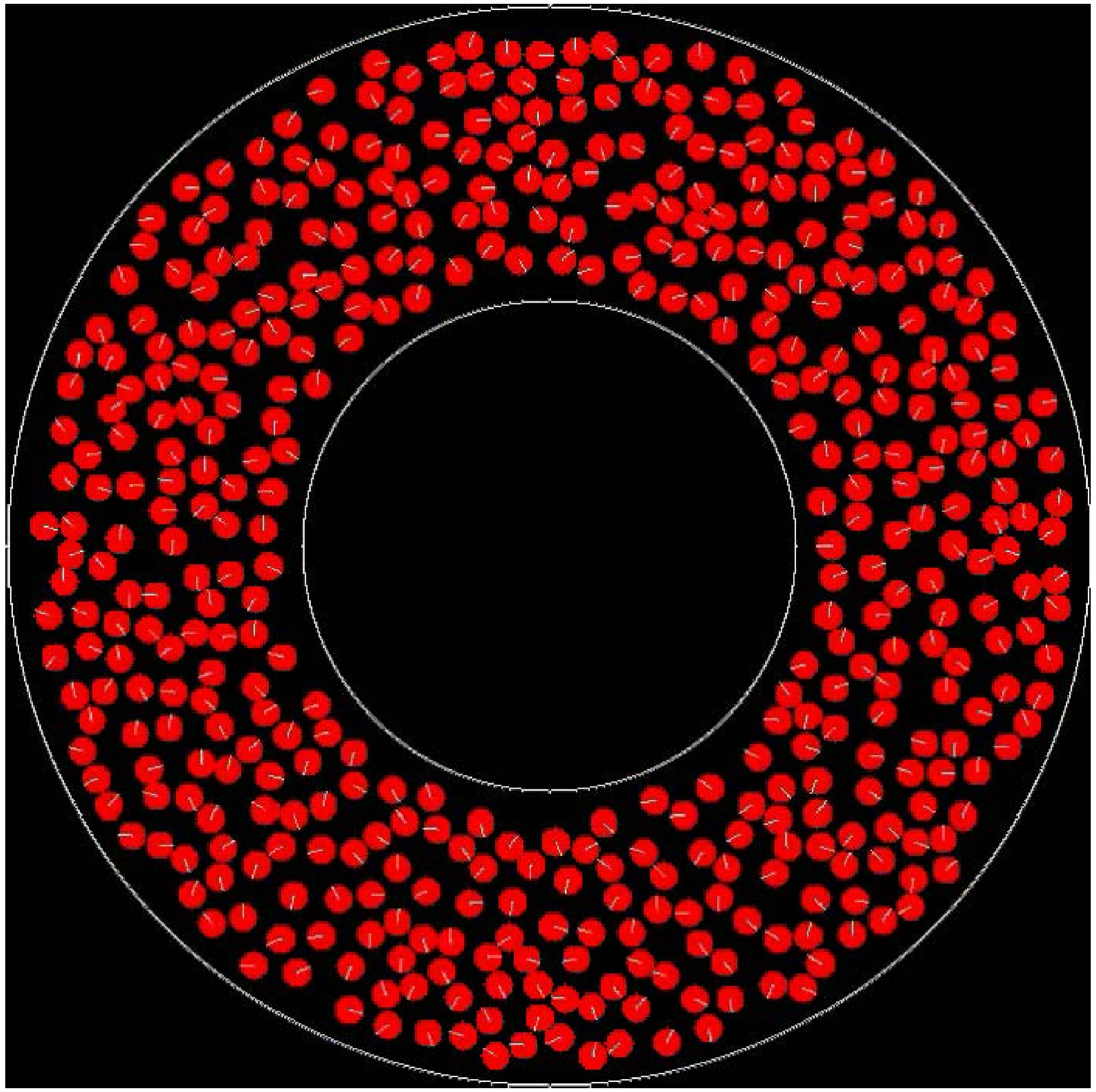
2. **Simple models** can reproduce this behavior

3. A **simple noise** term can account for numerous **complex deterministic** factors

4. **Role of border** is very different

5. In many cases ordering is due to motion! In other words: in SPP systems **momentum is not conserved!**





Simplest alignment model with hard core repulsion

Part II

Statistical mechanics of collective motion
(of SPP-s – i.e., self-propelled particles)

We are not in equilibrium, and even the momentum is not conserved!

still….

Order parameter is naturally expressed through the velocities

$$\varphi = \frac{1}{N v_0} \left| \sum_{i=1}^{N} \vec{v}_i \right|$$

As soon as we have an order parameter and the level of perturbations (analogous to the temperature in equilibrium statistical mechanics) expressions analogous to those in Eq. Stat. Mech. can be constructed and tested for validity.

Phase diagrams can be investigated.

For example, for the order parameter in Eq.S.M close to the critical temperature

$$\rho_l - \rho_g \sim (T_c - T)^\beta \qquad \xi \sim |\eta - \eta_c|^{-\nu}$$

While for SPP-s we usually write

$$\varphi \sim \begin{cases} (1 - \eta/\eta_c)^\beta & \text{for } \eta < \eta_c \\ 0 & \text{for } \eta > \eta_c \end{cases}$$

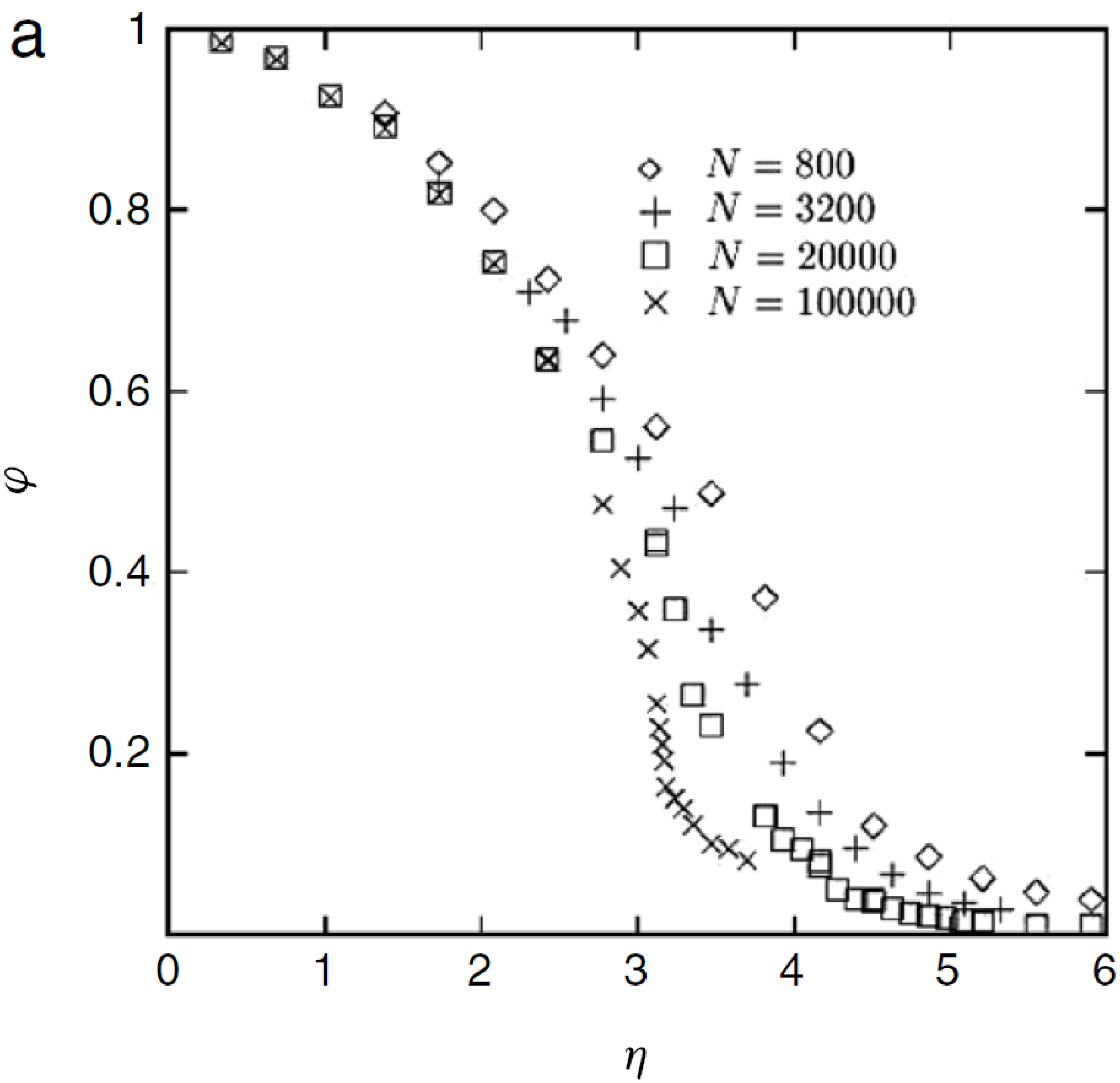
Velocity-velocity autocorrelation function

$$c_{vv}(t) = \frac{1}{N} \sum_{i=1}^{N} \frac{\langle \vec{v}_i(t) \cdot \vec{v}_i(0) \rangle}{\langle \vec{v}_i(0) \cdot \vec{v}_i(0) \rangle}$$

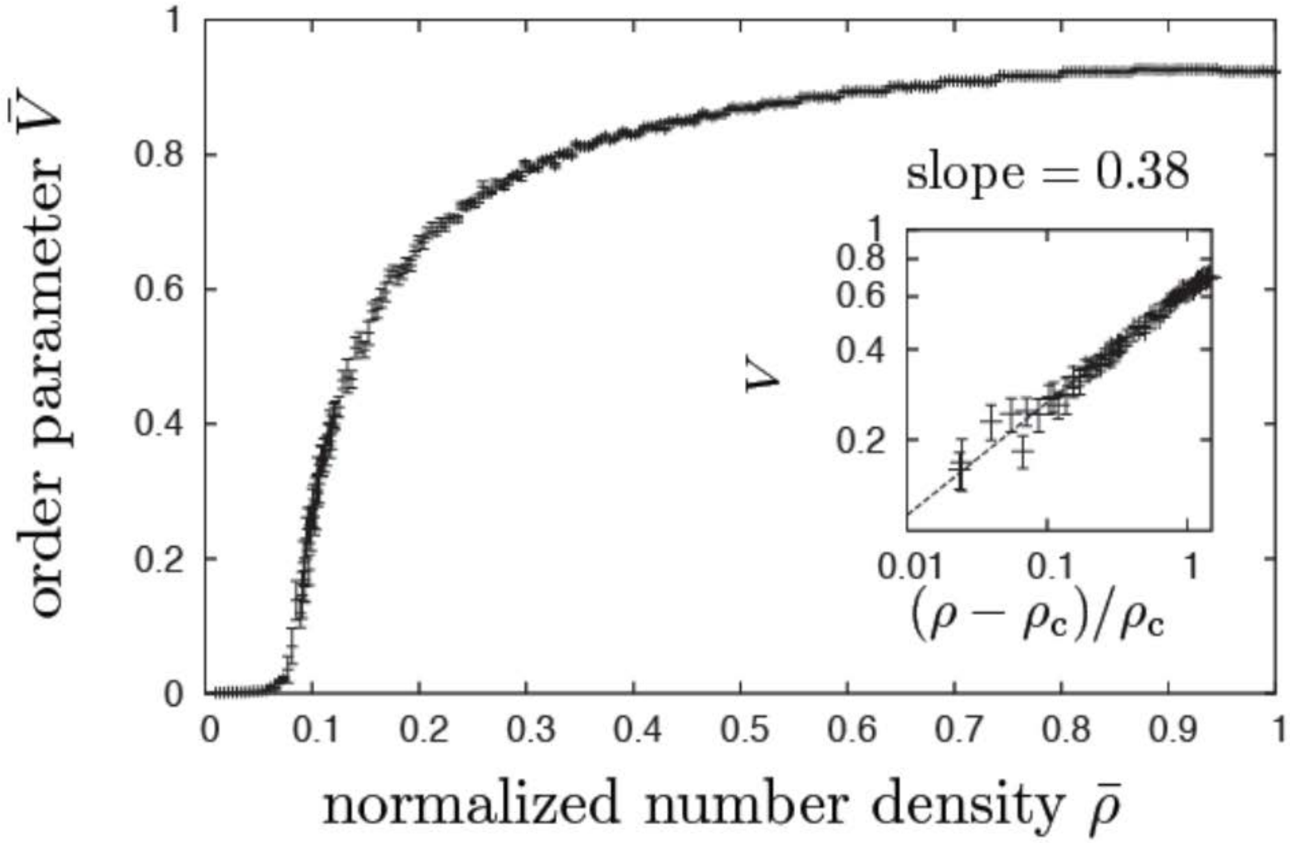
Directional correlation function

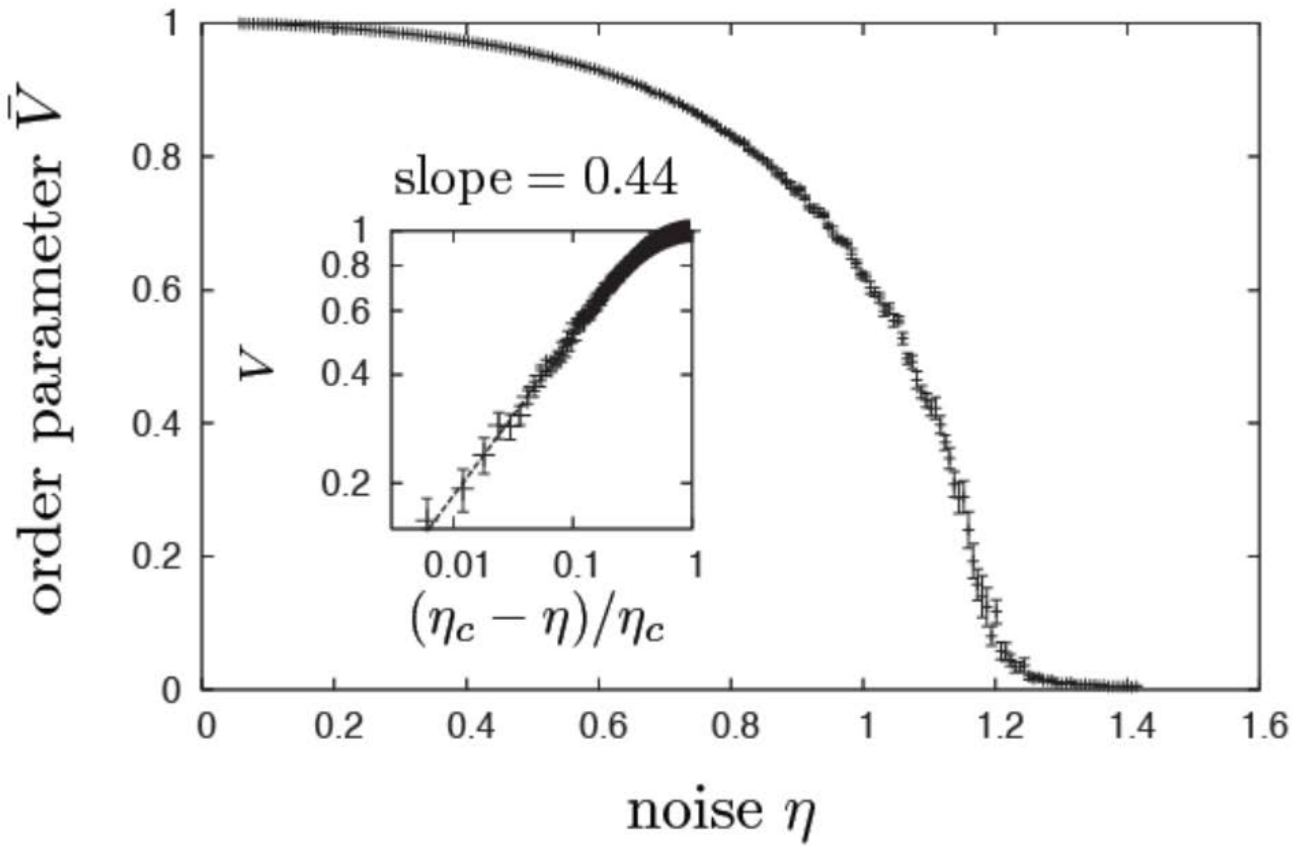
$$c_{ij}(\tau) = \left\langle \vec{v}_i(t) \cdot \vec{v}_j(t + \tau) \right\rangle$$

Order-disorder phase transition in the simplest alignment model for SPP-s (continuous)



For the soft push and adhere model continuous

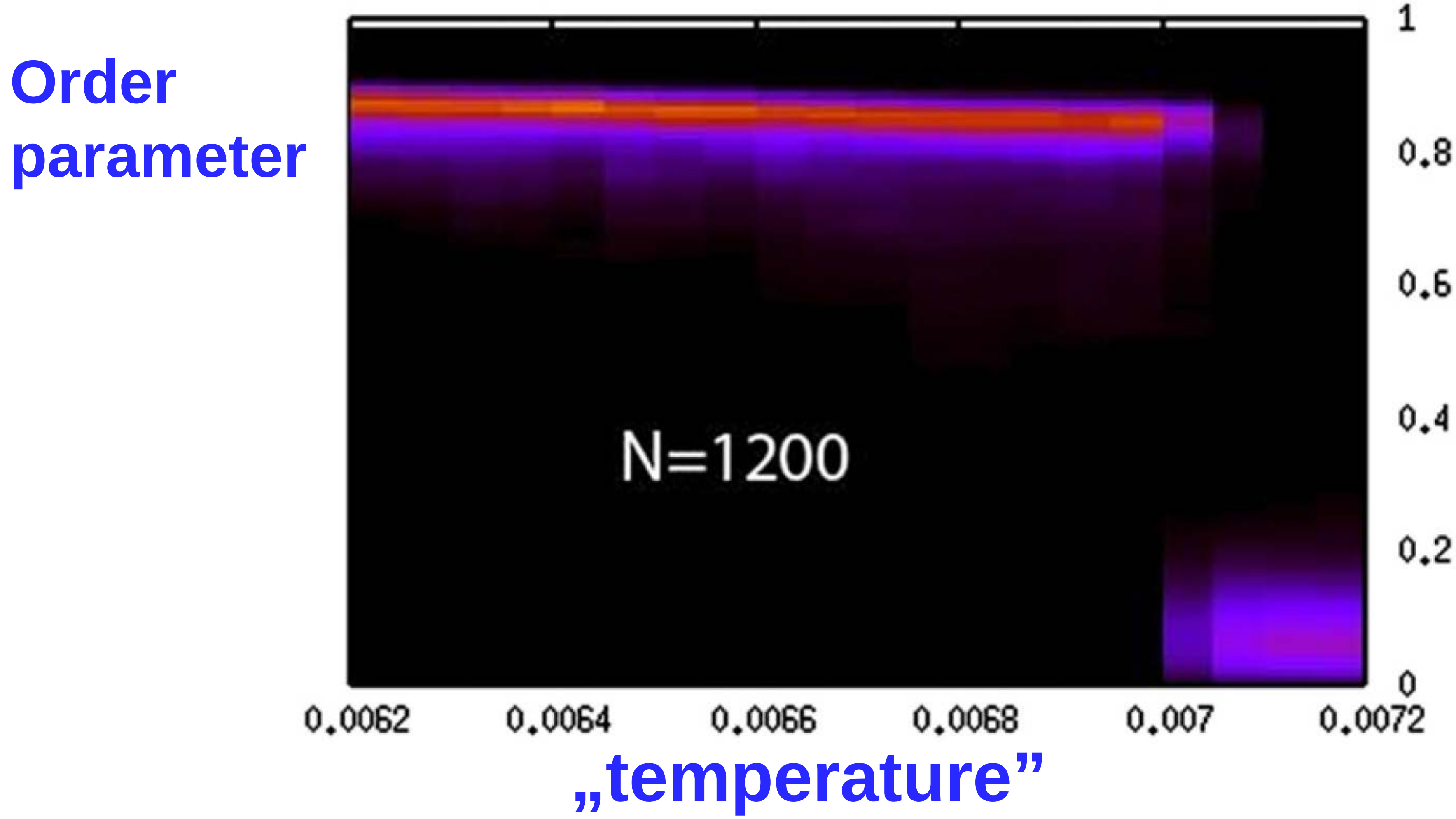


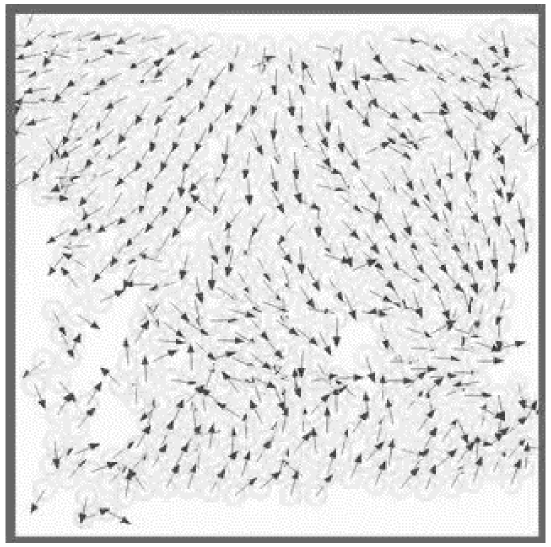


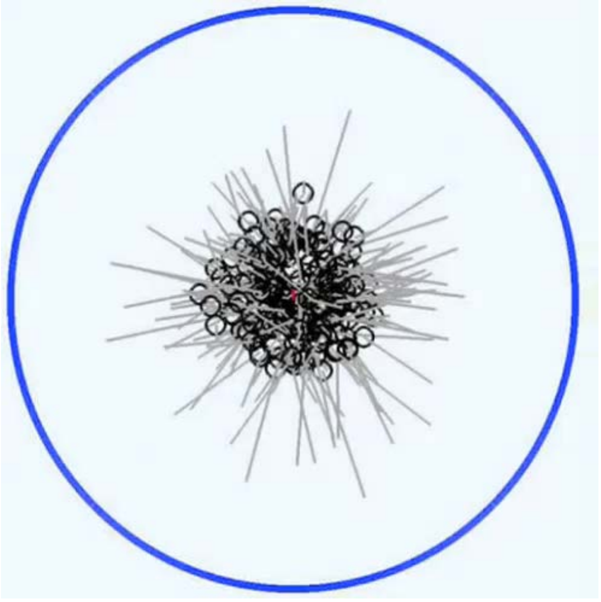
The order of phase transition

Previous plots: classic second order (continuous) for an alignment model (these are more common!)

The plot below: classic first order transition for a non-alignment model







Finite-size scaling: simple alignment SPP

Order parameter

$$\varphi(\eta, L) = L^{-\beta/\nu}\tilde{\varphi}((\eta - \eta_c)L^{1/\nu}),$$

$\xi \sim |\eta - \eta_c|^{-\nu}$

Susceptibility

$$\chi = \sigma^2 L^2 \qquad \sigma^2 \equiv \langle \varphi^2 \rangle - \langle \varphi \rangle^2$$

$$\chi(\eta, L) = L^{\gamma/\nu}\tilde{\chi}((\eta - \eta_c)L^{1/\nu})$$

Hyperscaling relation

$$d\nu - 2\beta = \gamma$$

e.g., $\tilde{\varphi}(x) \sim x^{\beta}$

For x >> 1 so that the order parmeter cannot be *L* dependent or

$\tilde{\varphi}(x) \sim$ *const*

For *x* << 0

Continuum equation of motion: Analogue of the Navier-Stokes for SPP

$$\partial_t \vec{v} + \lambda_1 (\vec{v}\nabla)\vec{v} + \lambda_2 (\nabla\vec{v})\vec{v} + \lambda_3 \nabla(|\vec{v}|^2) =$$

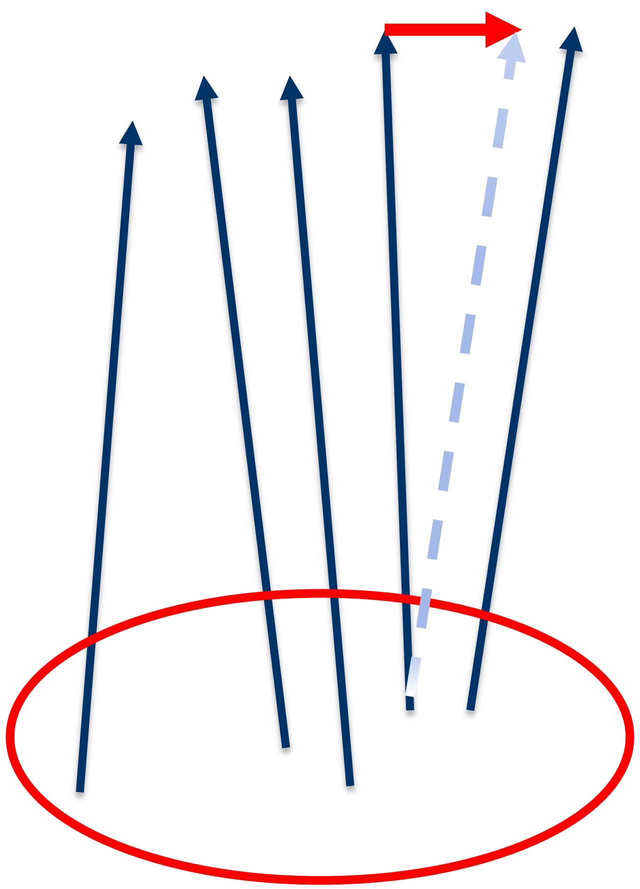
$$= \alpha\vec{v} - \beta|\vec{v}|^2\vec{v} - \nabla P +$$

$$+ D_L \nabla(\nabla\vec{v}) + D_1 \nabla^2 \vec{v} + D_2 (\vec{v}\nabla)^2 \vec{v} + \vec{\xi}$$

$$\partial_t \rho + \nabla(\rho\vec{v}) = 0.$$ Conservation of mass

$$P = P(\rho) = \sum_{n=1}^{\infty} \sigma_n (\rho - \rho_0)^n$$

No Galilean invariance!



„perpendicular" perturbations are stronger „noise" acts on them

Treatment by dynamic renormalization group or Numerical integration.

Scaling of the directional correlations is found close to the critical noise

Can collective cell migration enhance cell segregation?

In vitro system:

- **Mixed co-culture**
- **No prepattern**
    - **Differential adhesion**

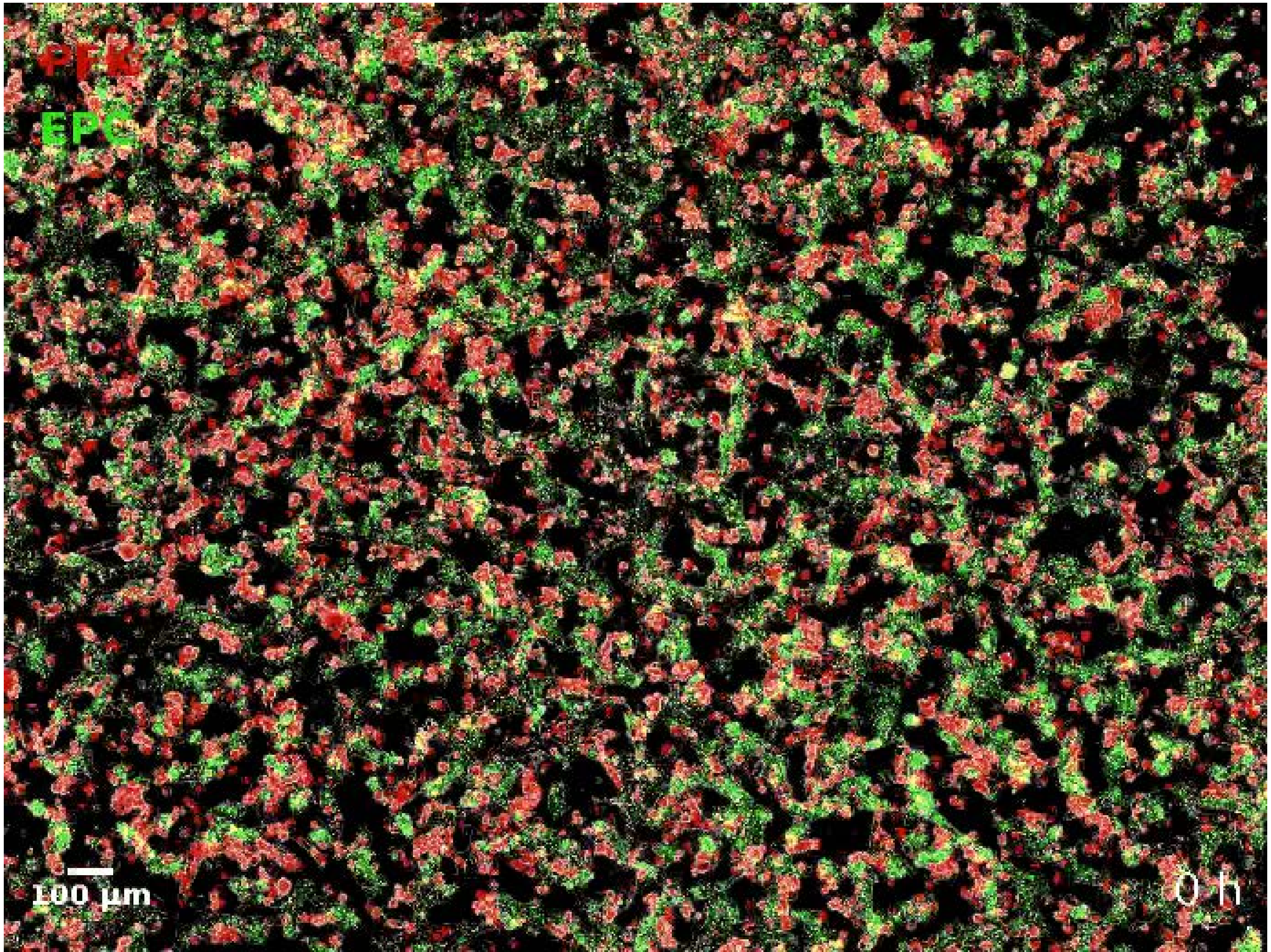
A → A   B → B

A ↛ B

Anomalous segregation

**for a model introduced for cells in Part I,
but zero adhesion between cells of different kinds,
„Red” and „Green” and a stronger adhesion
between the Green cells**

Experiment:

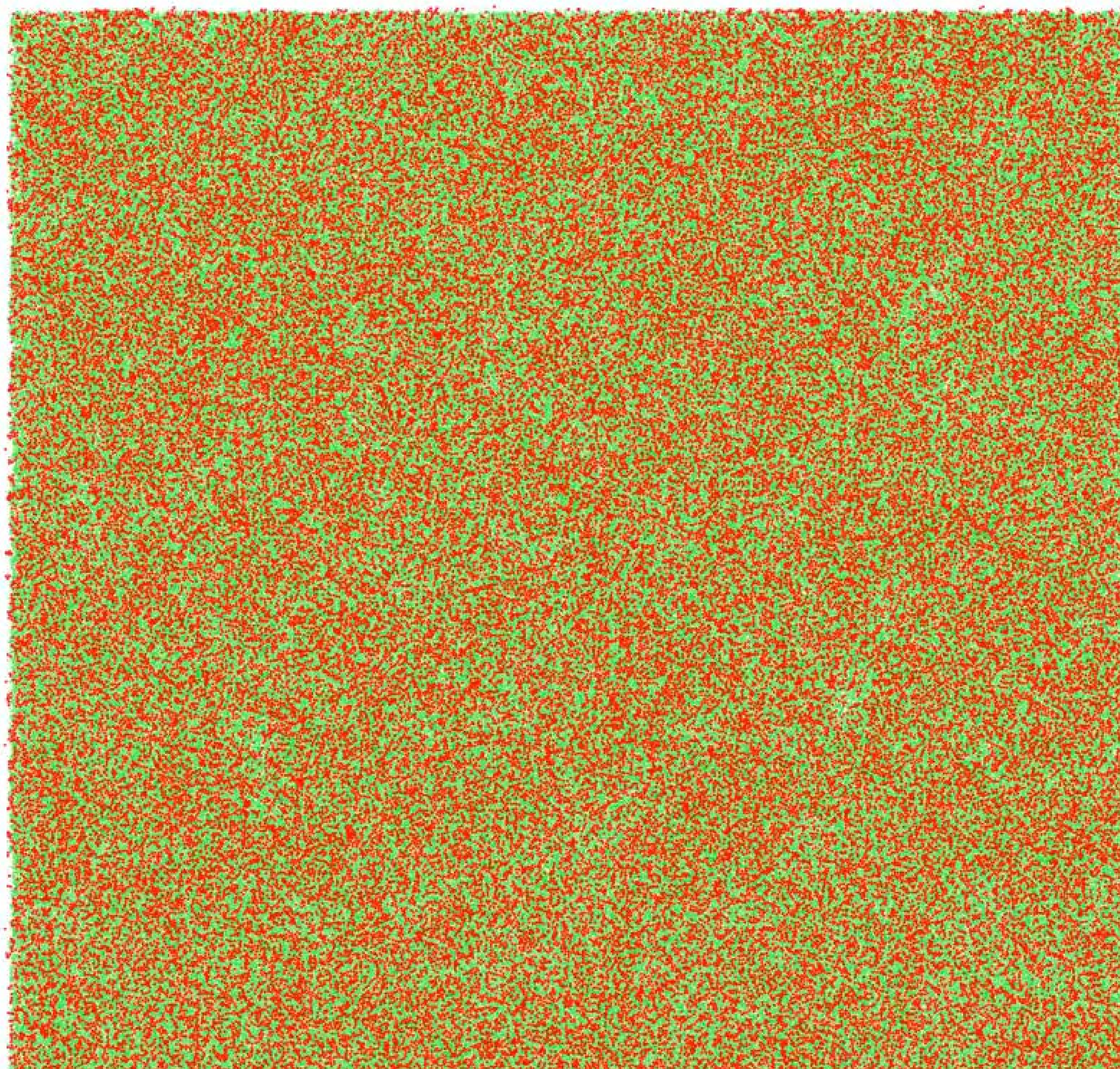


For Brownian

$$\lambda \sim t^z$$

$z = 1/3$ for even coverage ratios

$z = 1/4$ for unequal



For SPP-s

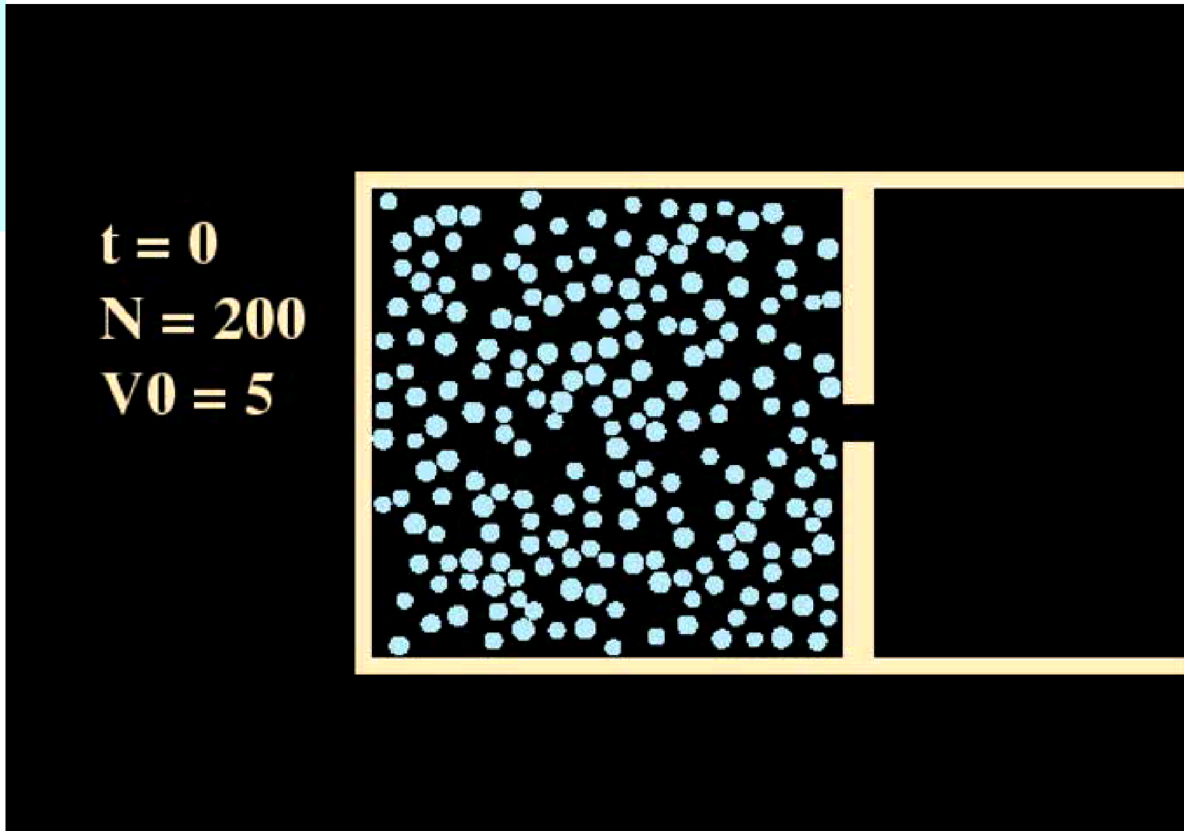
$z = 1$

Motion of people in a crowd satisfies Newton's equations of motion

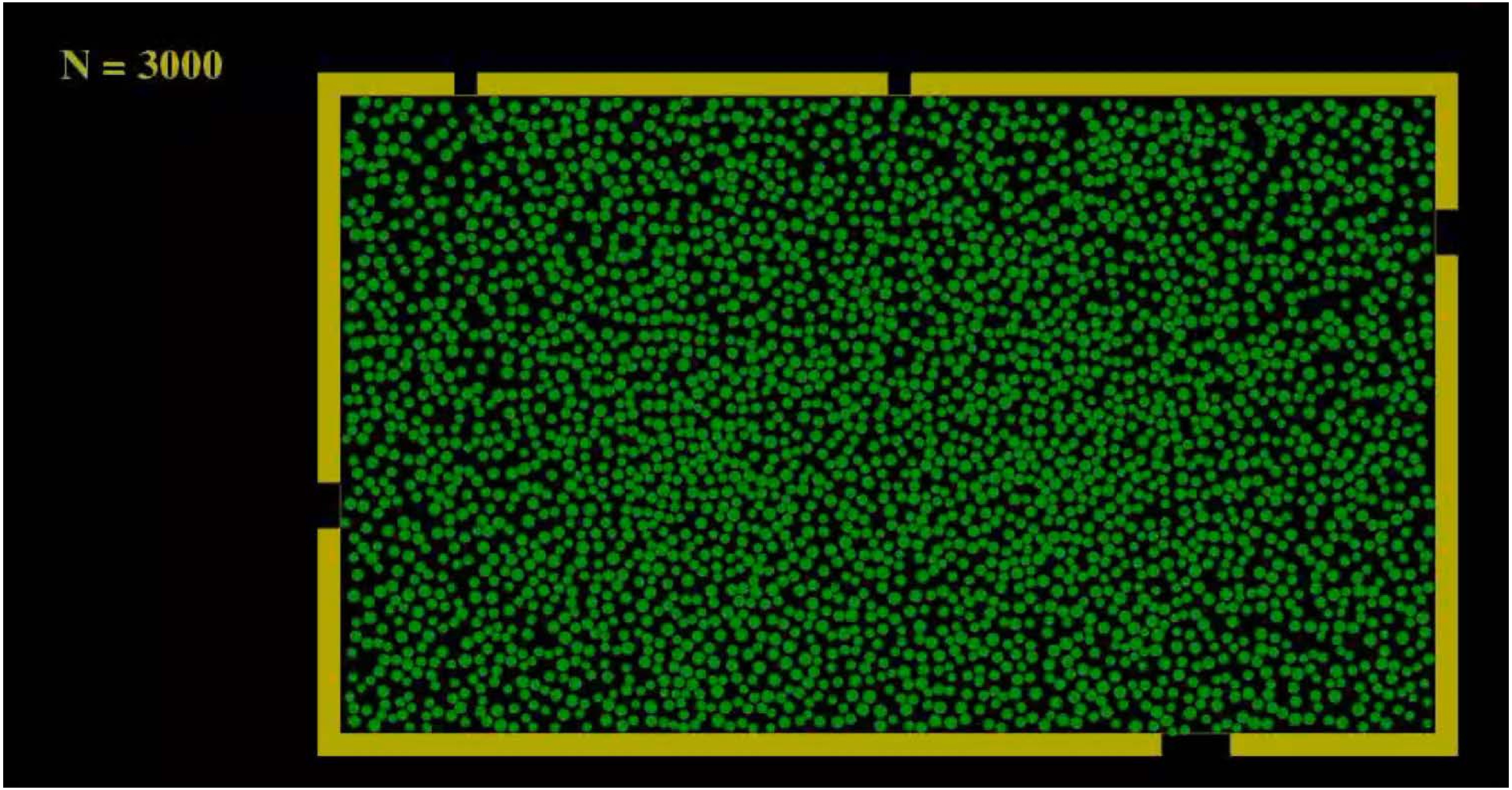
$$m_i \frac{d\vec{v}_i}{dt} = m_i \frac{v_i^0(t)\vec{e}_i^{\,0}(t) - \vec{v}_i(t)}{\tau_i} + \sum_{j \neq i} \vec{f}_{ij} + \vec{f}_{iW} \quad ,$$

$$\vec{f}_{ij} = \left[A_i \exp\left[\left(r_{ij} - d_{ij} \right) / B_i \right] + kg\left(r_{ij} - d_{ij} \right) \right] \vec{n}_{ij} + \kappa g\left(r_{ij} - d_{ij} \right) \Delta v_{ji}^t \vec{t}_{ij} \quad ,$$

$$\vec{e}_i^{\,0}(t+1) = N\left[\left(1 - p_i\right) \vec{e}_i(t) + p_i \left\langle \vec{e}_j(t) \right\rangle_j \right] ,$$



Escape: several doors, unpatient



Colour codes the level of pressure

Universal classes of flocking patterns ("phases")

i) *disordered* (particles moving in random directions)

ii) *fully ordered* (particles moving in the same direction)

iii) *rotational* (within a rectangular or circular area)

vi) *critical* (flocks of all sizes moving coherently in different directions. The whole system is very sensitive to perturbations)

v) *Jamming*

Plus several more exotic phases

Collective landing of flocks

SPP flocking rules in quasi 2d (horizontally)

+

„landing rules” vertically

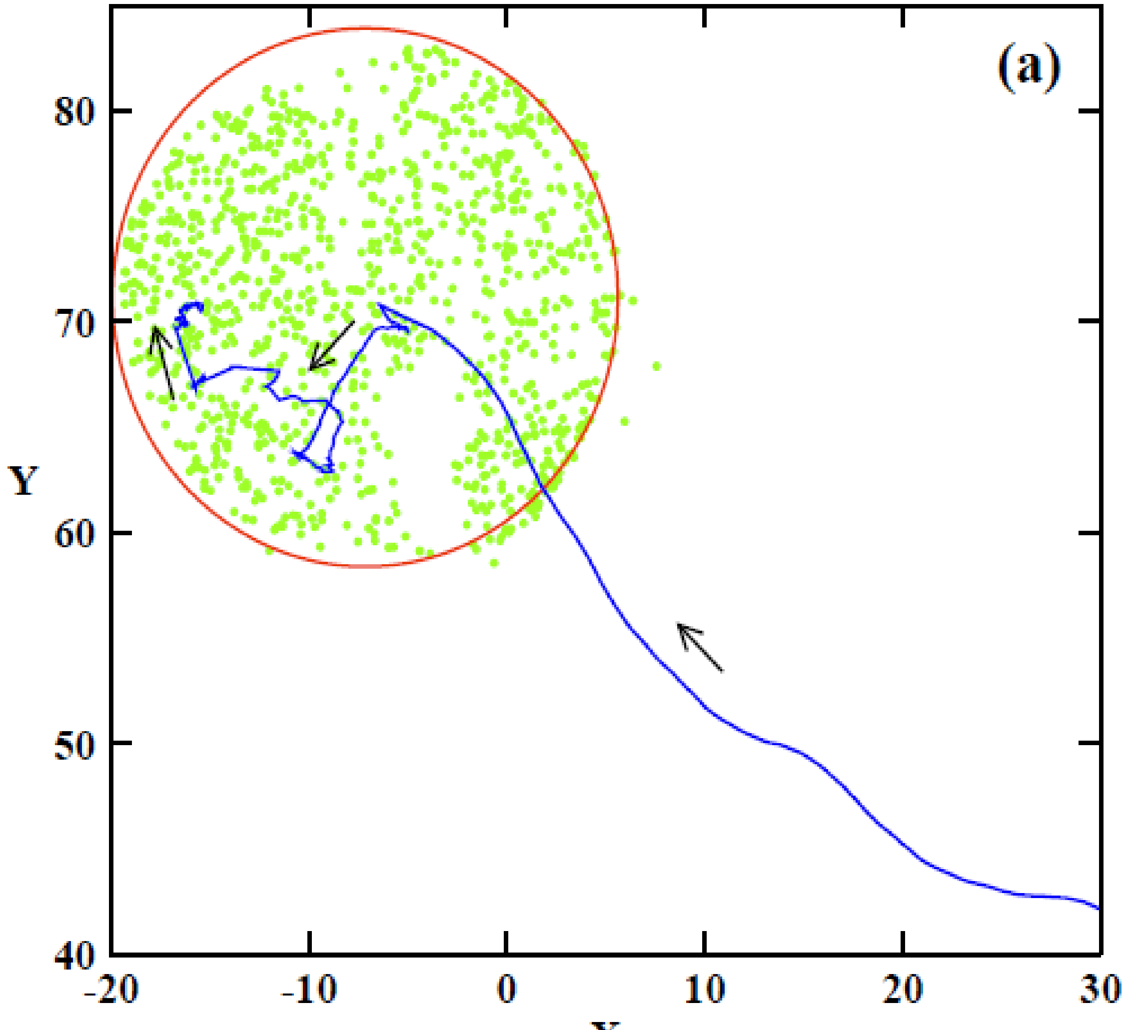
„Collective landing” here stands for (a paradigm of) a group decision on a simultaneous starting or stopping of an activity

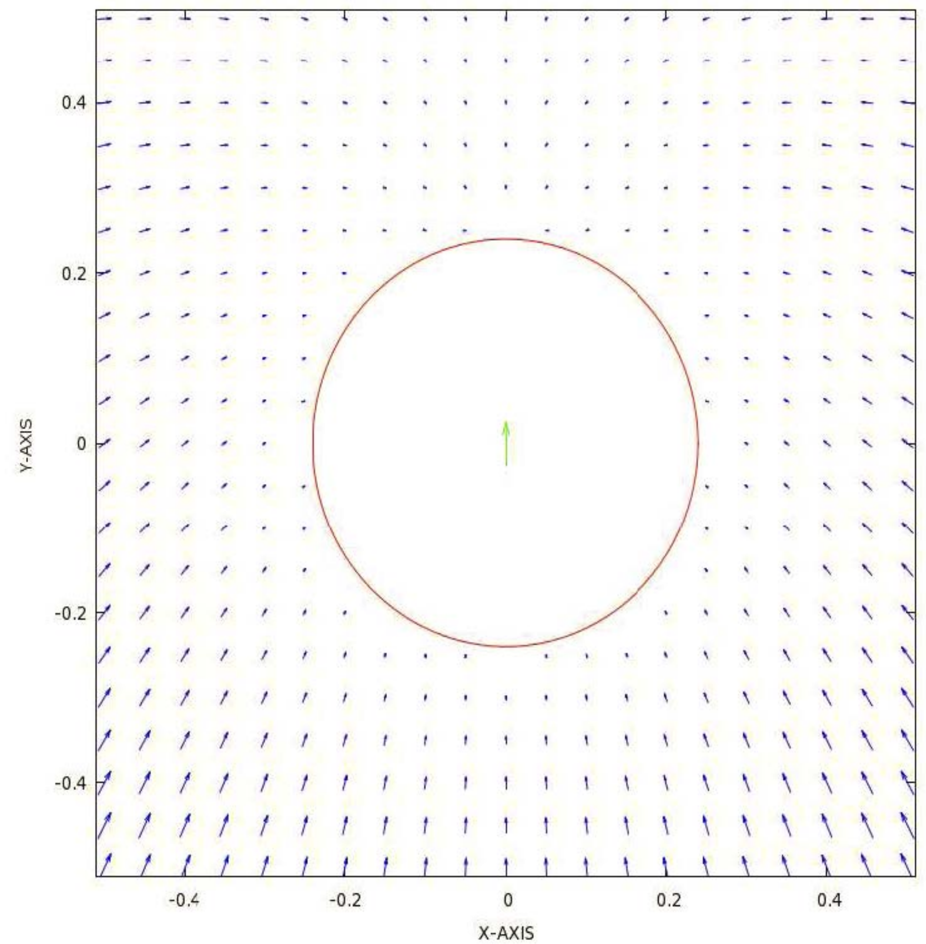
Vertical interaction: RFIM (random field Ising model) type

(i.e., birds have bias towards the „decisions" of their neighbours)

- increasing tiredness -> locally growing external field
- two states: moving upward or downward

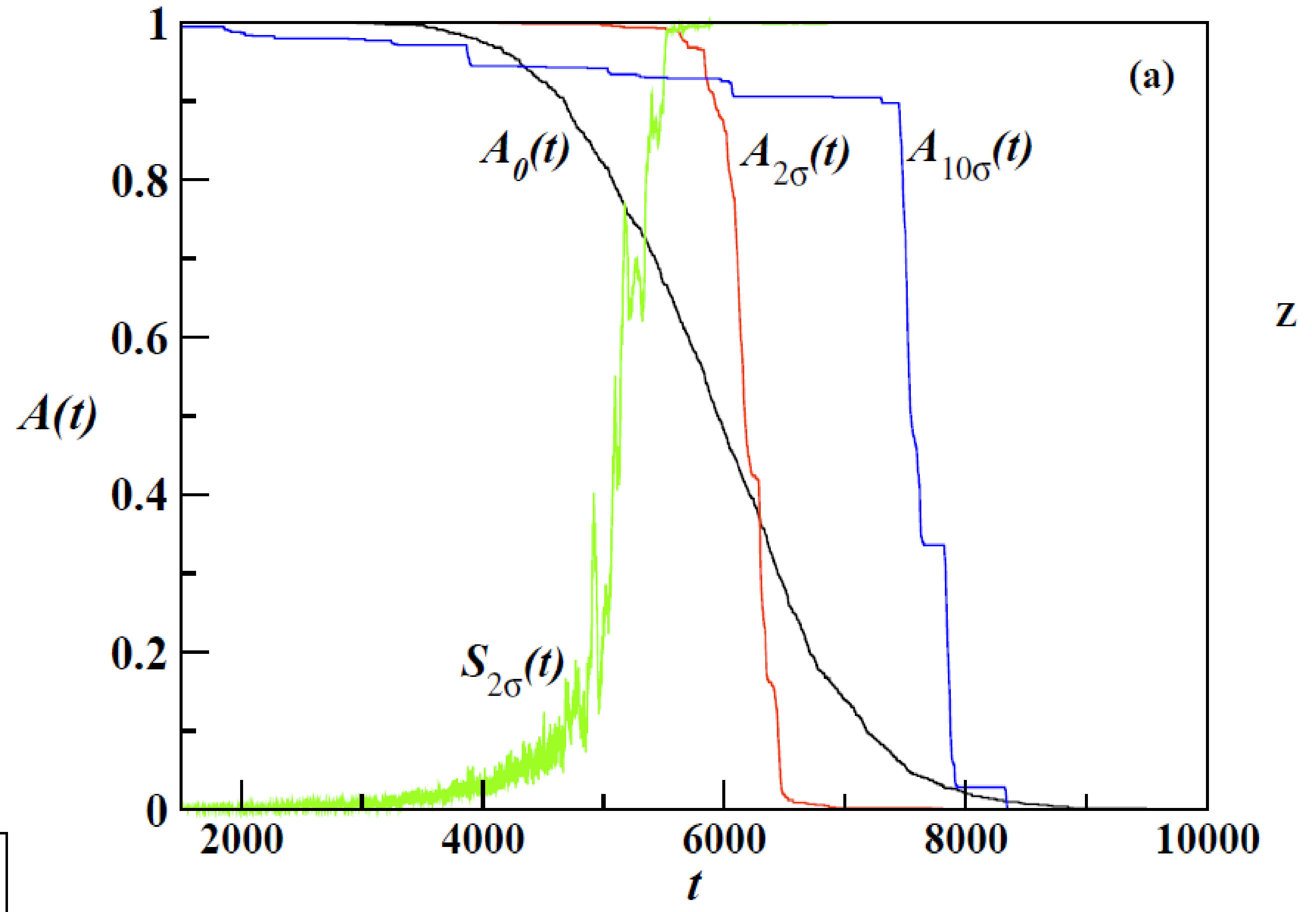
An appropriate co-moving boundary condition is needed!

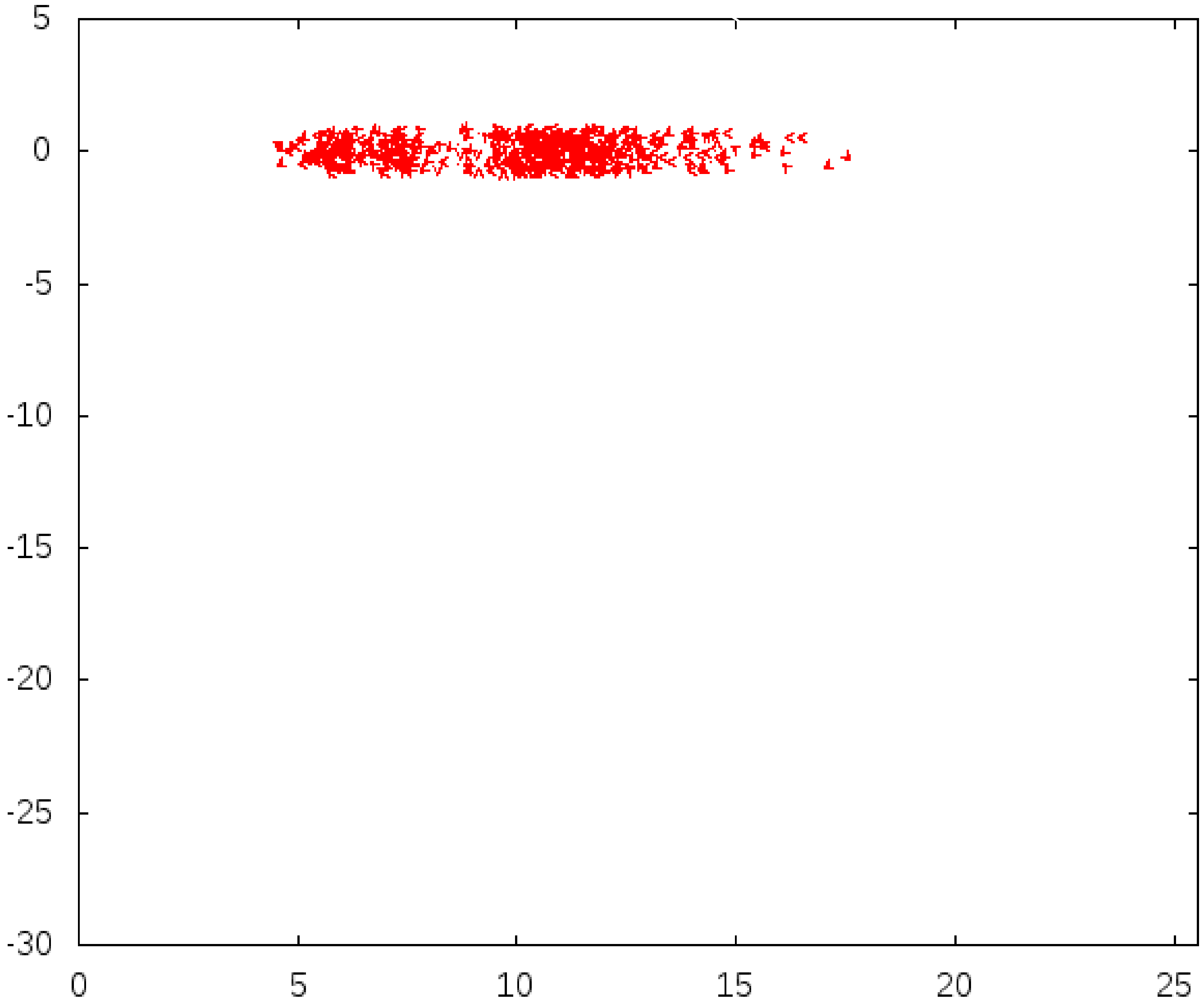




Results

σ is the strength of coupling in the vertical direction





We assume:

- The birds are getting tired (and make a move downward) but rather un-evenly
- They are motivated to stay with the others move back up if not followed
- If the majority of their neighbours decide to land, they land

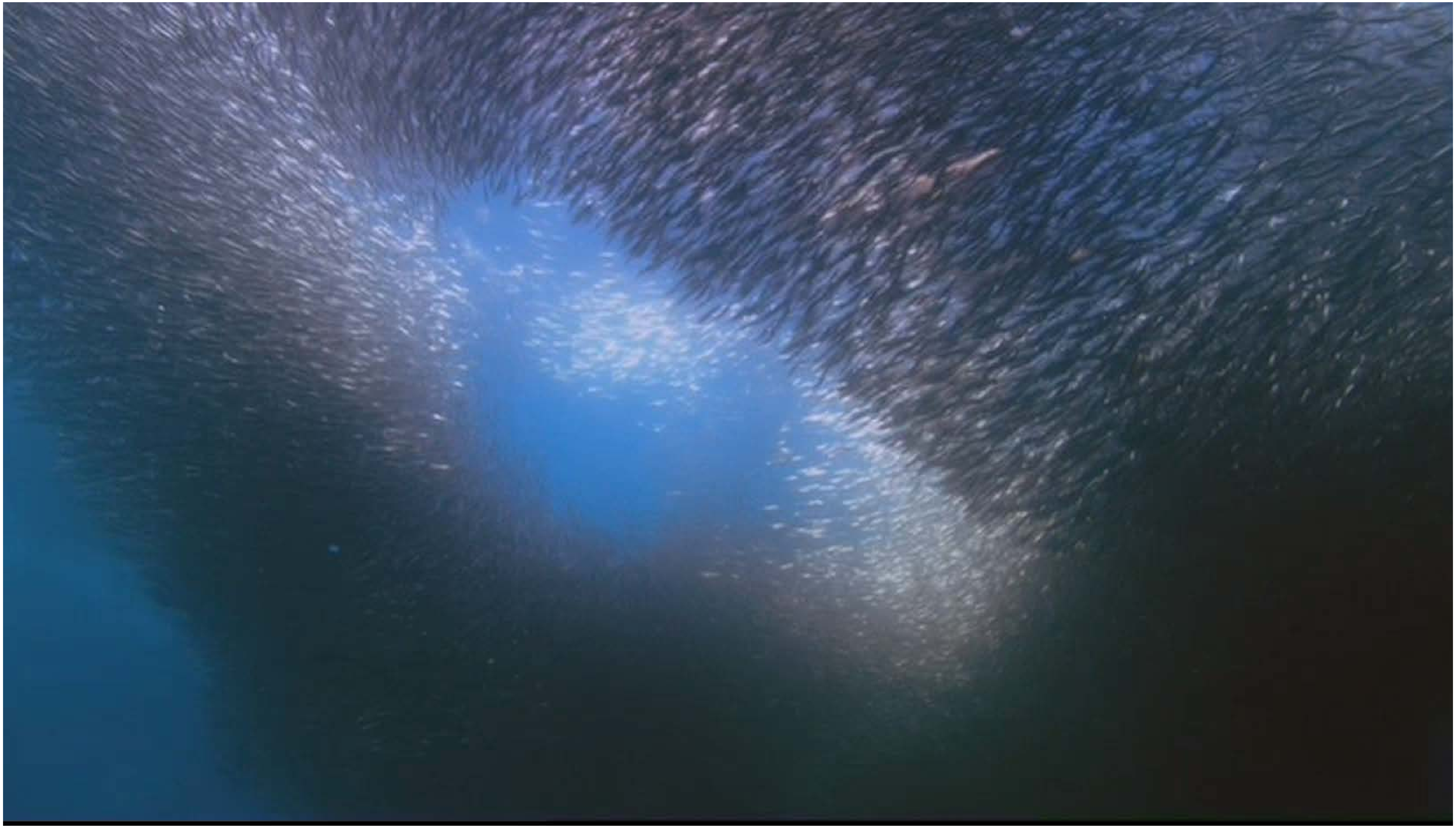
PART III

Group decision-making on the move: selected applications

- The physics of group hunting (realistic simulation)

- Hiearachical leadership/dominance in pigeon flocks

- Flocking drones (quadcopters)

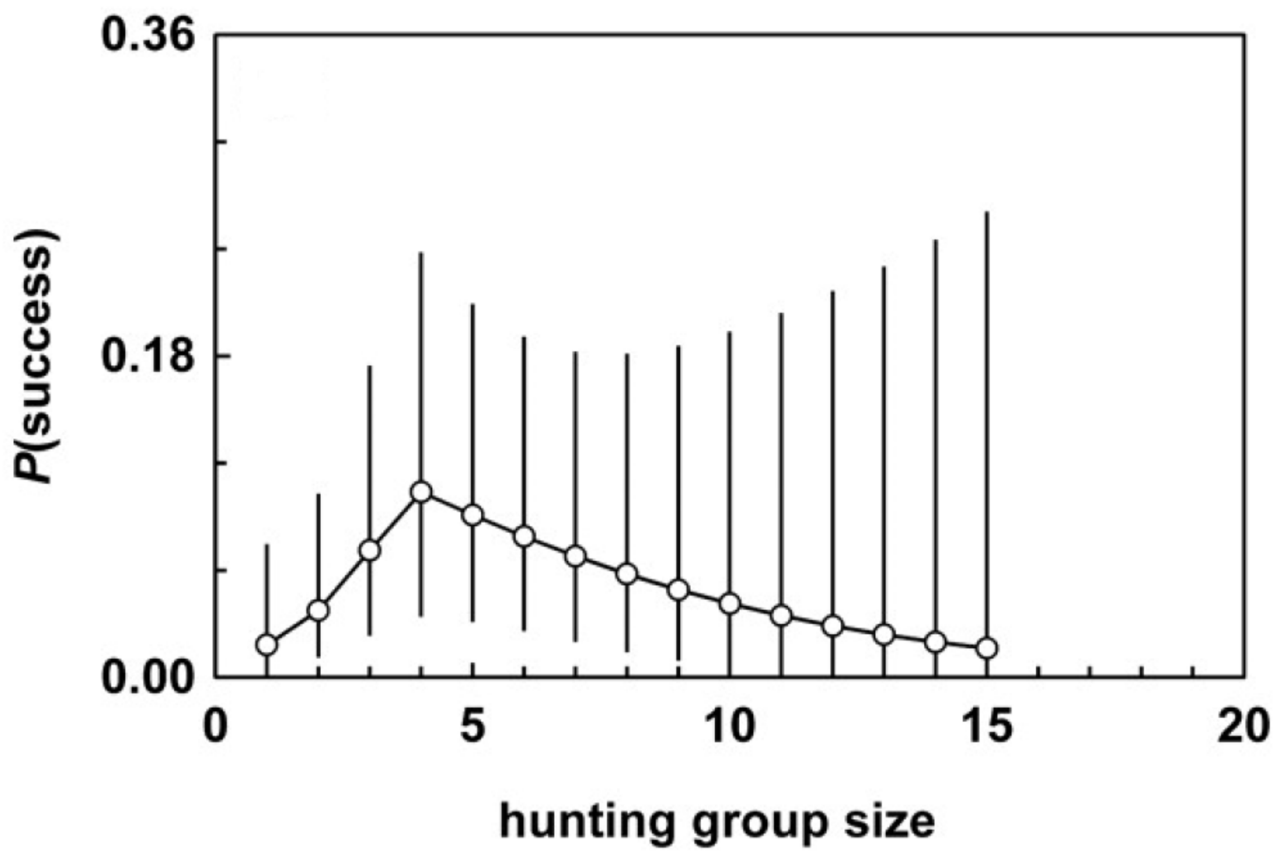


Several slower predators chase faster prey(s)
The case of collective hunting

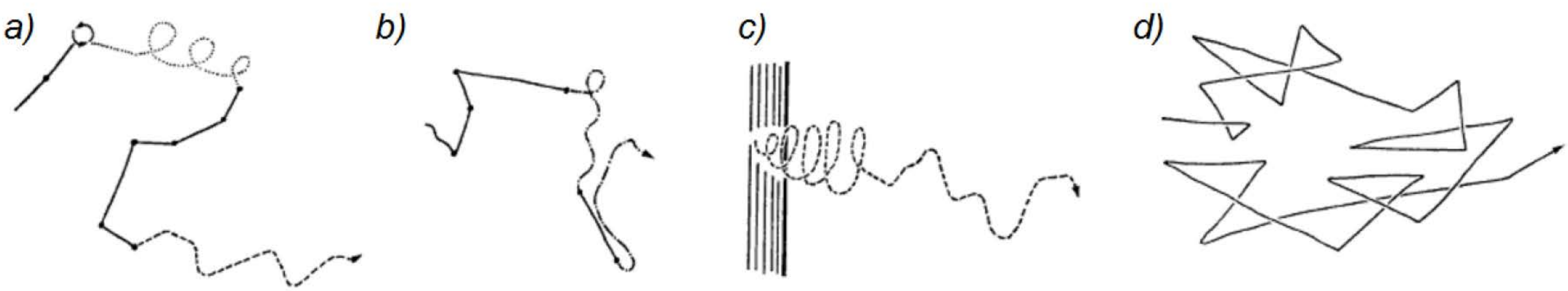
A complex set of equations, taking into account:

- **Instantaneous velocities**
- **Collision avoidance**
- **Predicted positions**
- **Delayed reactions**
- **Perturbations**
- **Boundary conditions**
- **Escaping tactics („zig-zag” running)**
- **Otimizing the parameters**
- **Etc.**

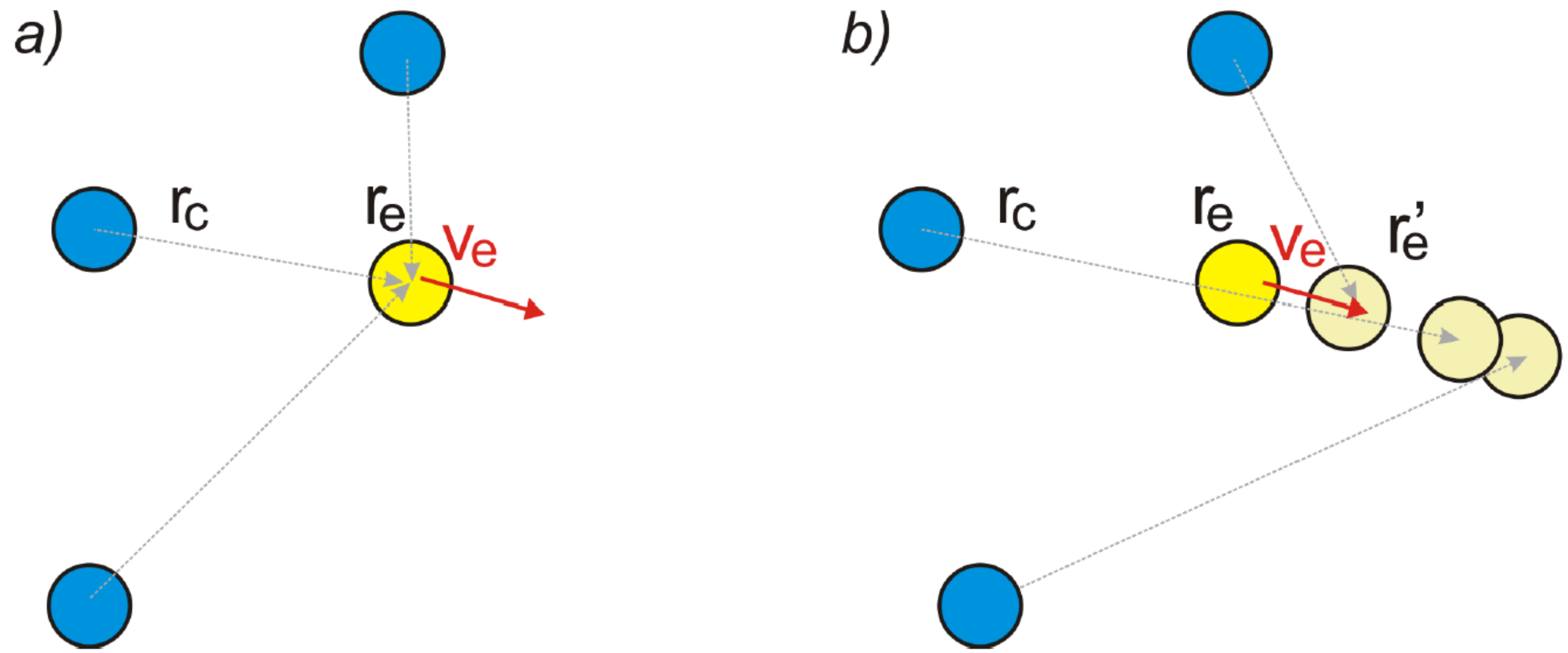
Wolf pack versus elk

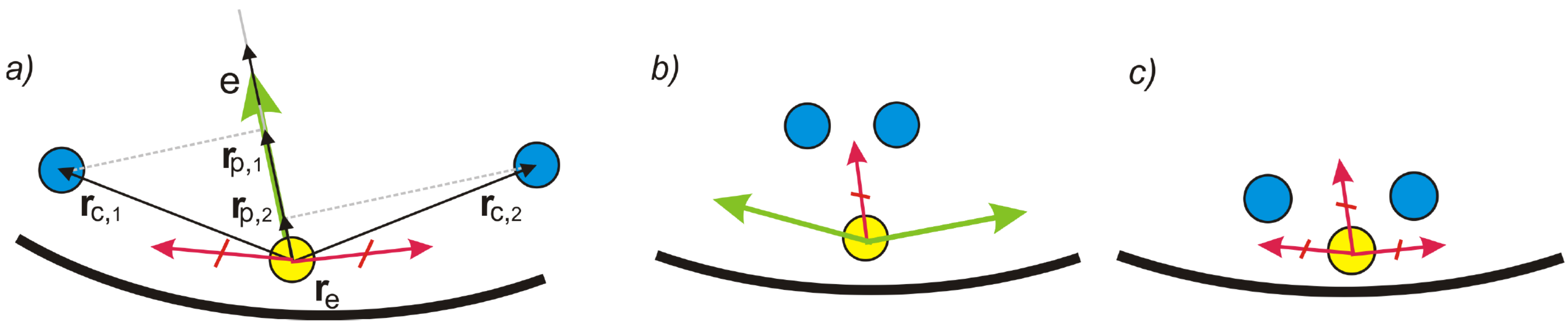


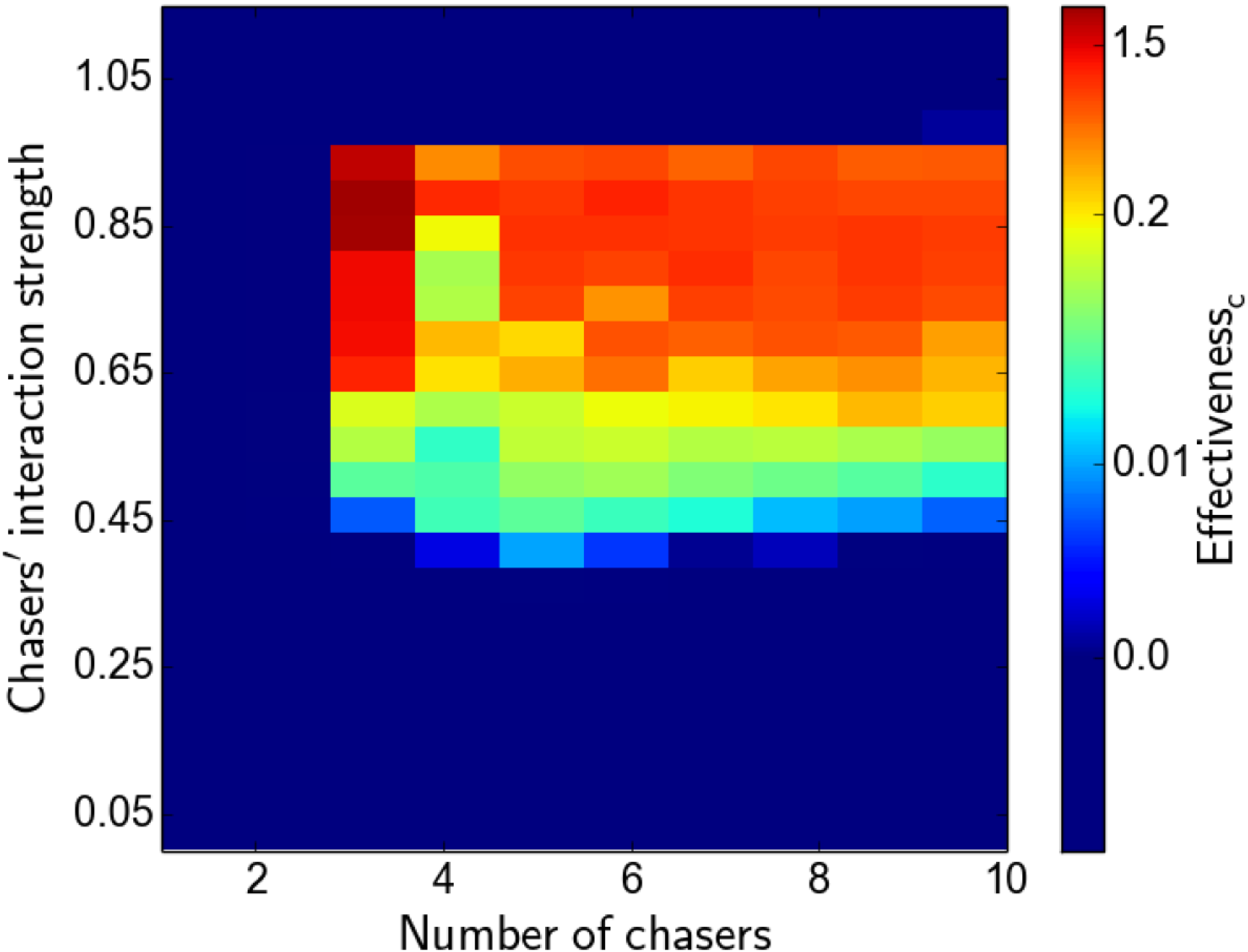
Escape tactics (trajectories)

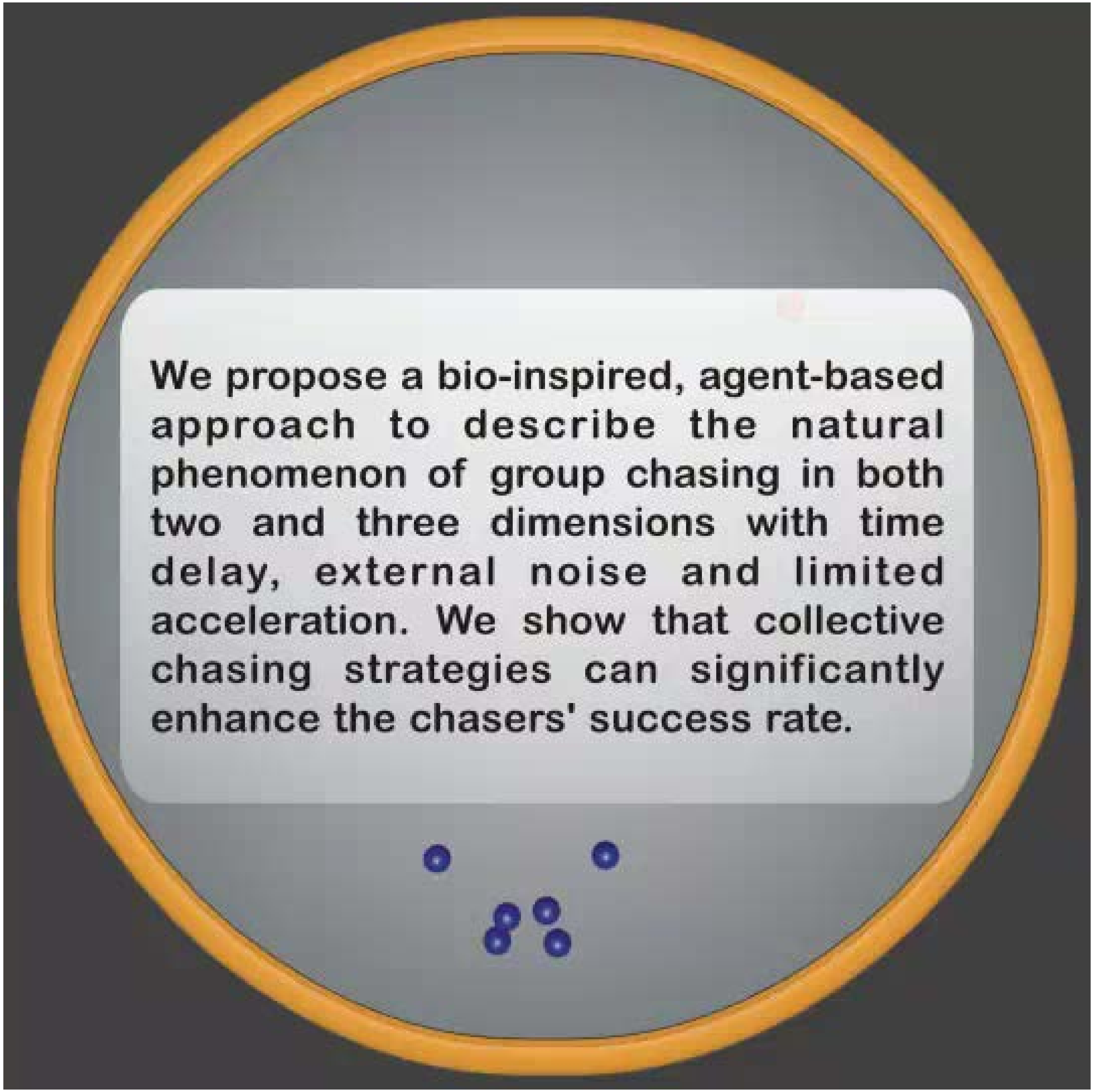


Example: prediction





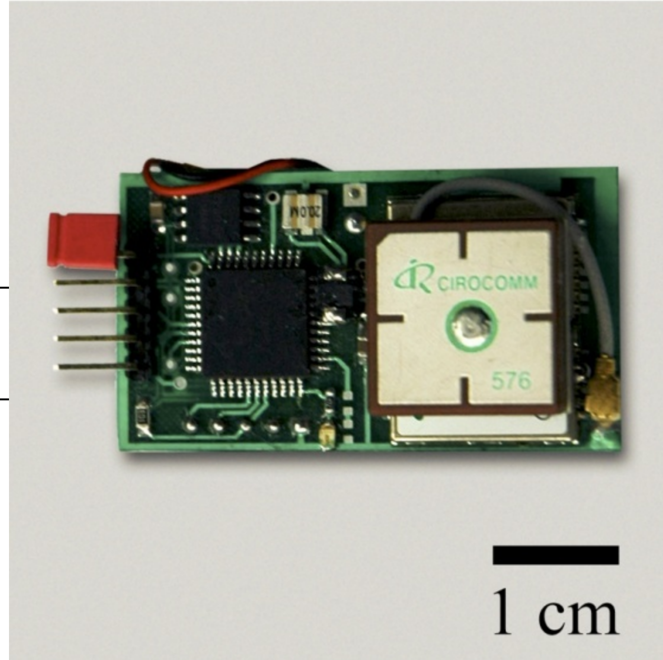

1.05
0.85
0.65
0.45
0.25
0.05
Chasers' interaction strength
2
4
6
8
10
Number of chasers
1.5
0.2
0.01
0.0
Effectiveness$_c$


We propose a bio-inspired, agent-based approach to describe the natural phenomenon of group chasing in both two and three dimensions with time delay, external noise and limited acceleration. We show that collective chasing strategies can significantly enhance the chasers' success rate.

Hierarchical group dynamics in pigeon flocks

A group of homing pigeons: paradigm of making collective decisions about choosing the right answer

Studies of pigeon flocks have a history

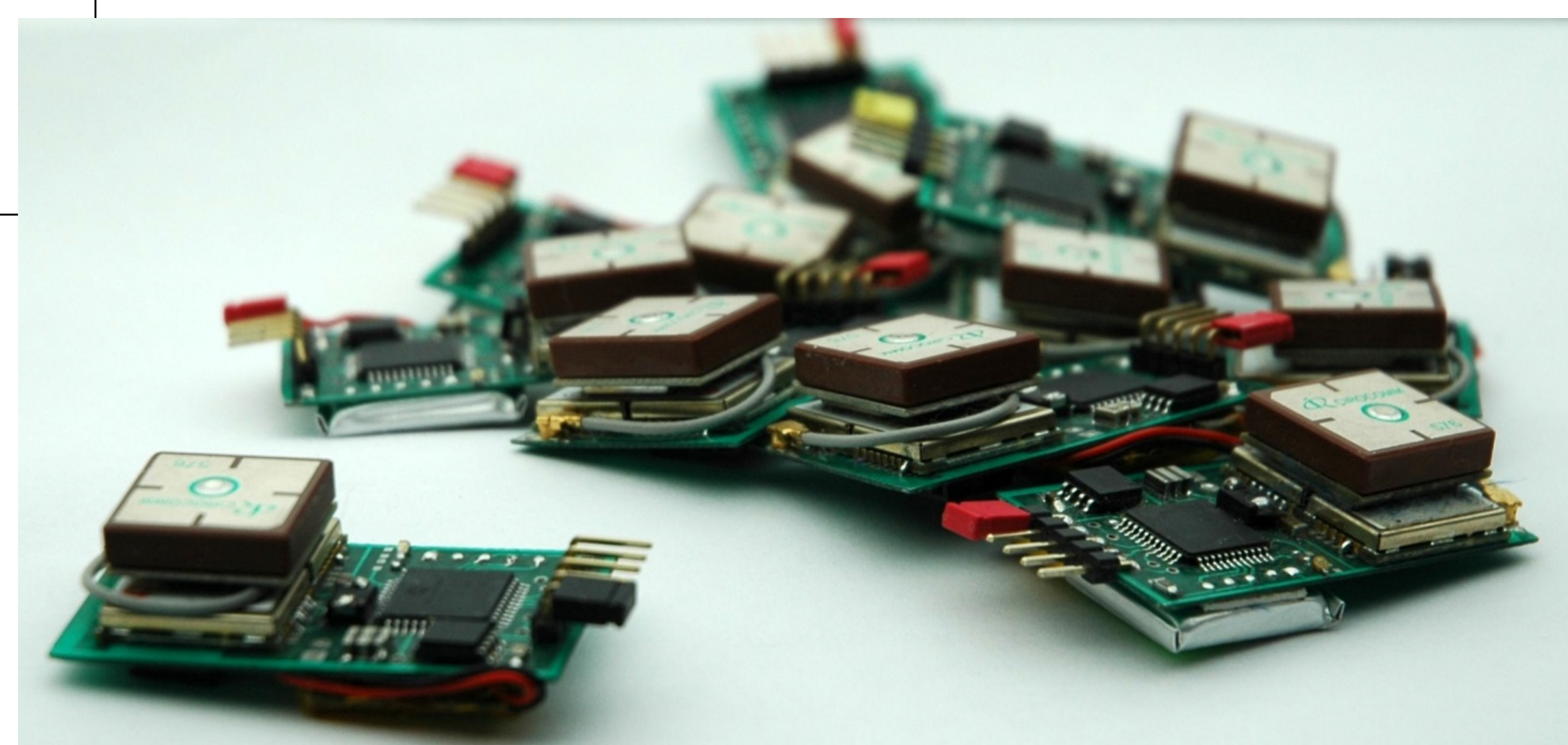


GPS module: Switzerland, U-blox,
(17 X 22 mm, 2,1g), 5Hz (2,5 Hz)

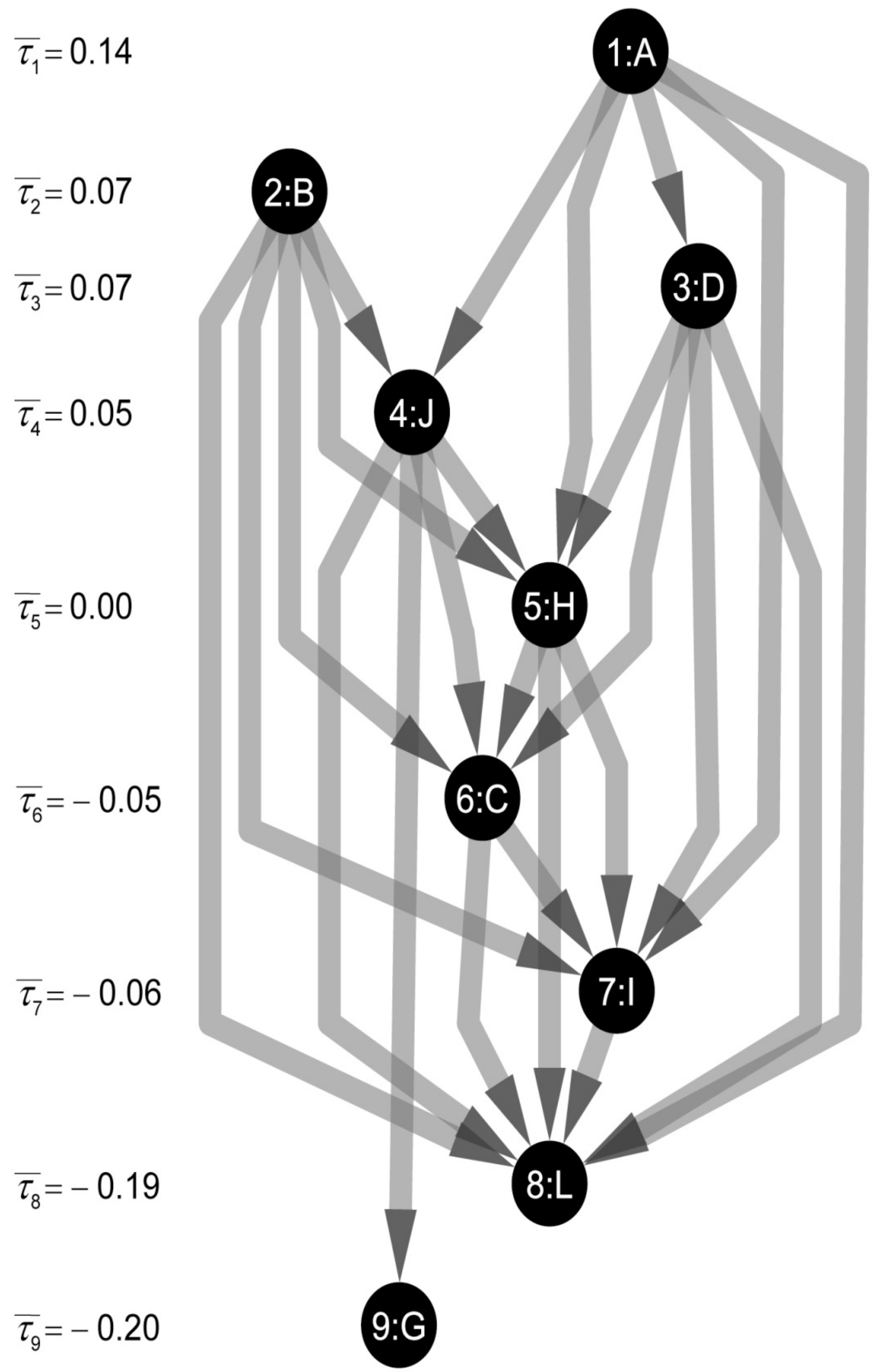
antenna, Ireland, Taoglas

accumulator : lipoly 2,9g (100mAh)

Weight: 13g

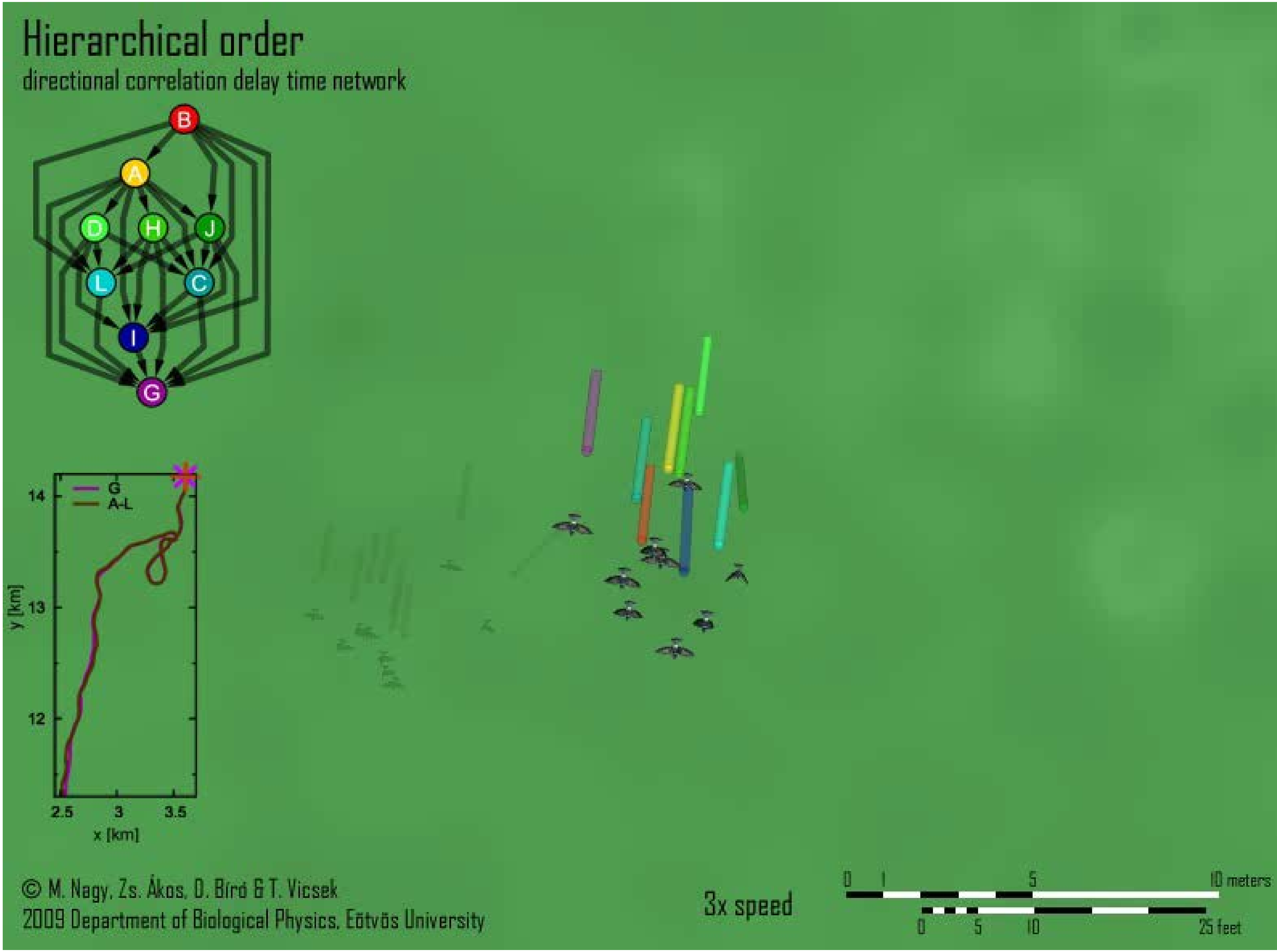


A



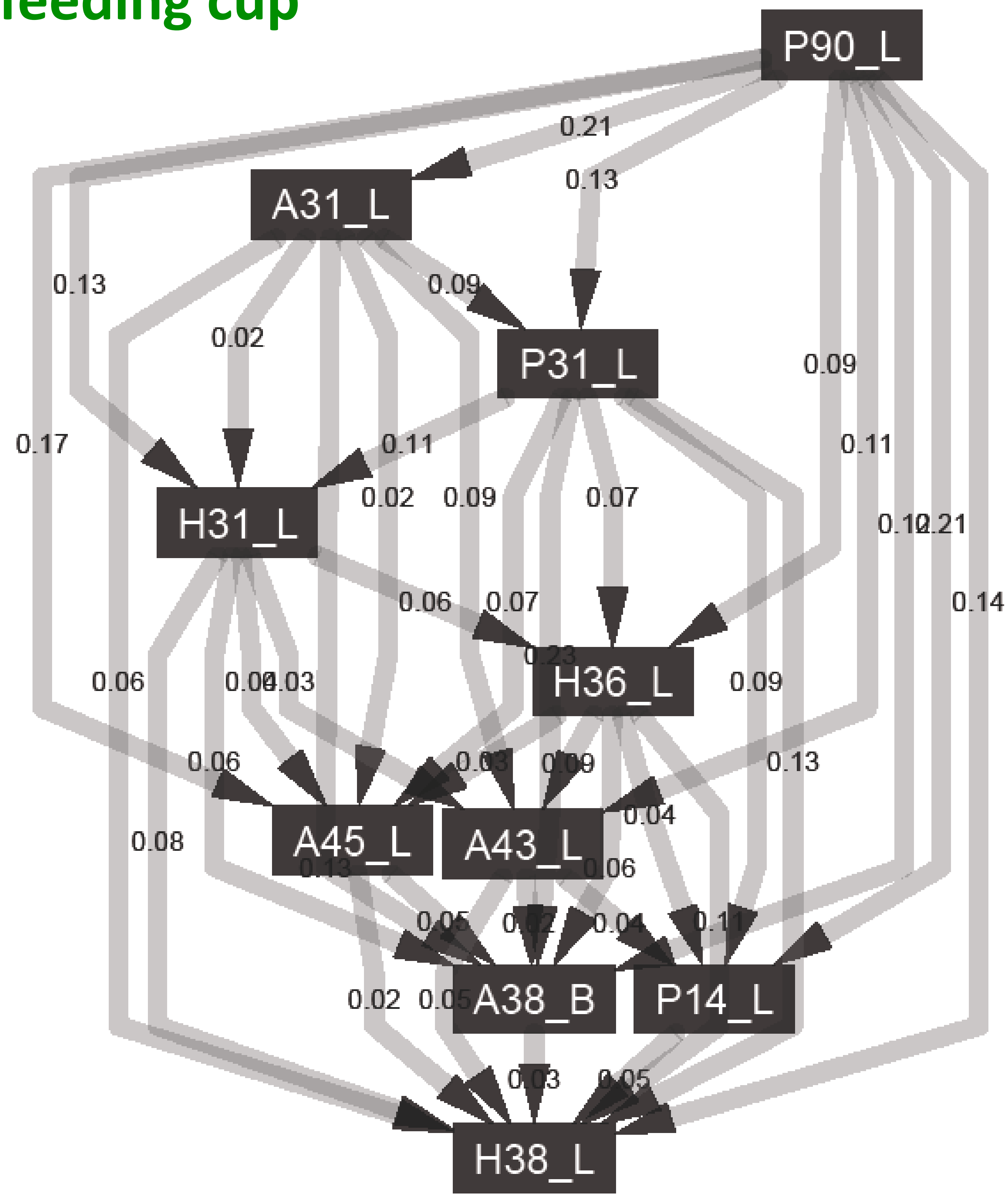
$\overline{\tau}_1 = 0.14$
$\overline{\tau}_2 = 0.07$
$\overline{\tau}_3 = 0.07$
$\overline{\tau}_4 = 0.05$
$\overline{\tau}_5 = 0.00$
$\overline{\tau}_6 = -0.05$
$\overline{\tau}_7 = -0.06$
$\overline{\tau}_8 = -0.19$
$\overline{\tau}_9 = -0.20$
1:A
2:B
3:D
4:J
5:H
6:C
7:I
8:L
9:G

B


Hierarchical order
directional correlation delay time network
B
A
D
H
J
L
C
I
G
G
A-L
14
13
12
y [km]
2.5
3
3.5
x [km]
© M. Nagy, Zs. Ákos, D. Bíró & T. Vicsek
2009 Department of Biological Physics, Eötvös University
3x speed
0
1
5
10 meters
0
5
10
25 feet

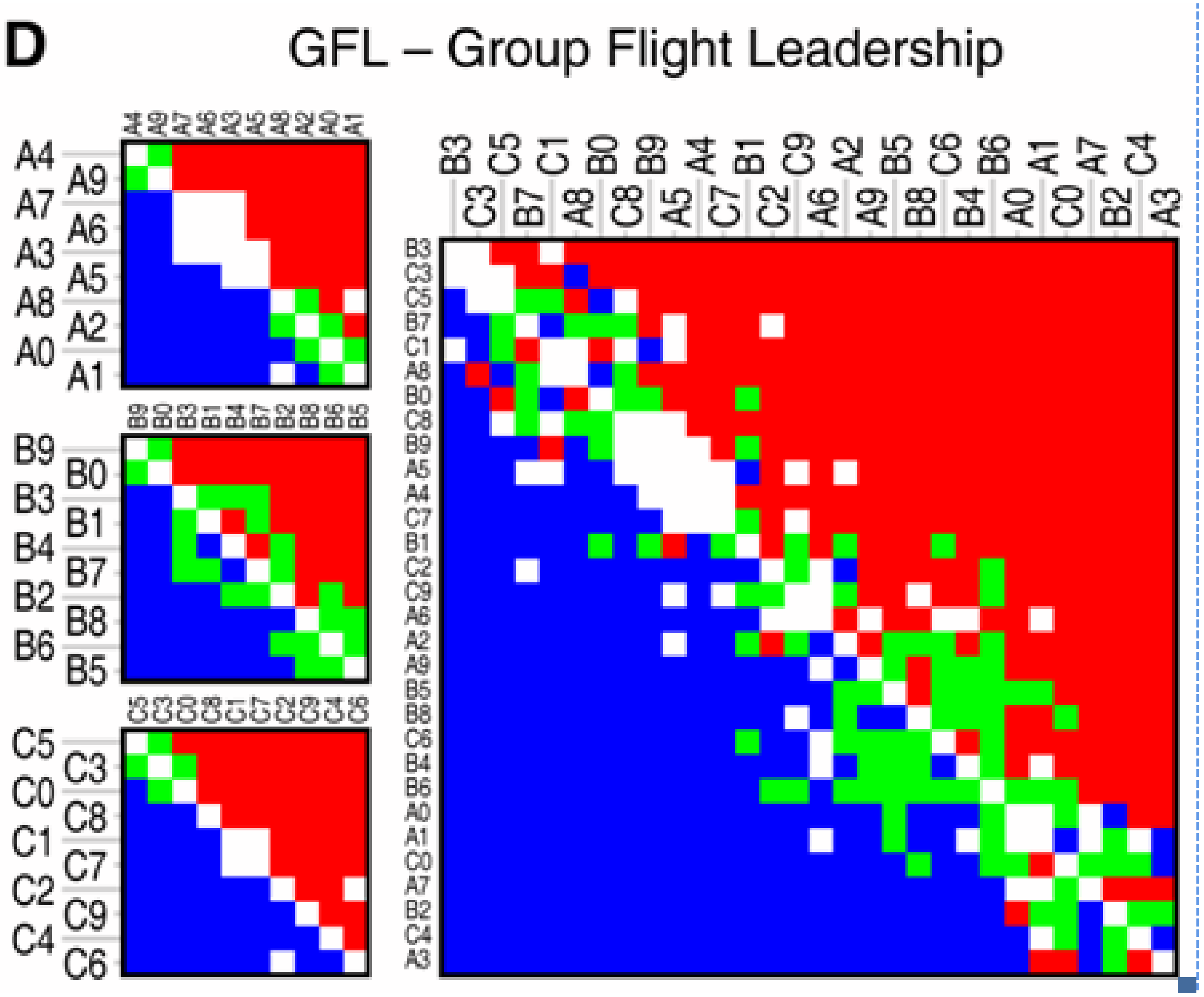
Digital video analysis of the moving pigeons around the feeding cup

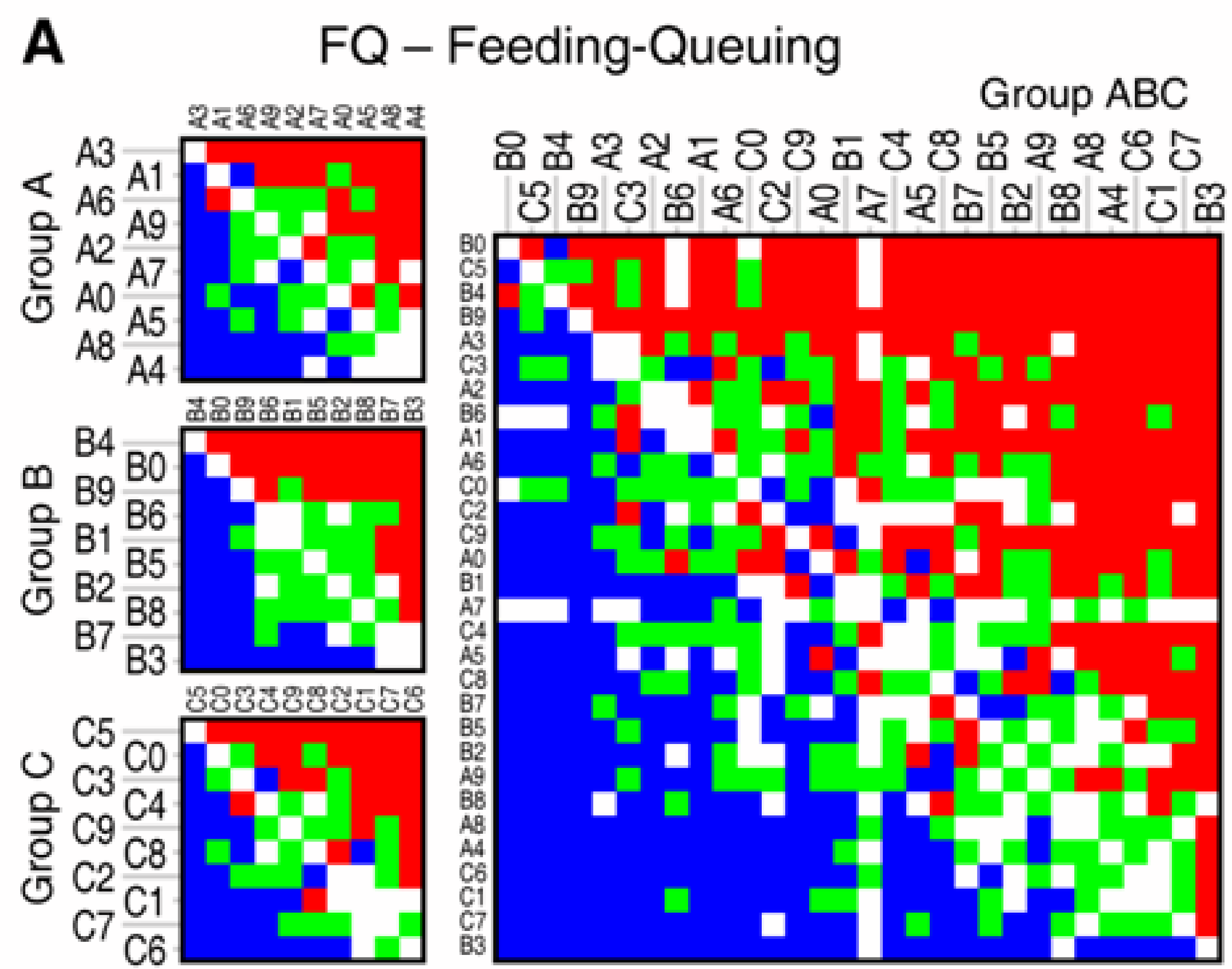
Pair-wise dominance graph as determined from „who is closer to the feeding cup”



Correlation of interaction matrices is nearly zero:

For pigeons the knowledge-based and the dominance hierarchies are <u>independent</u>





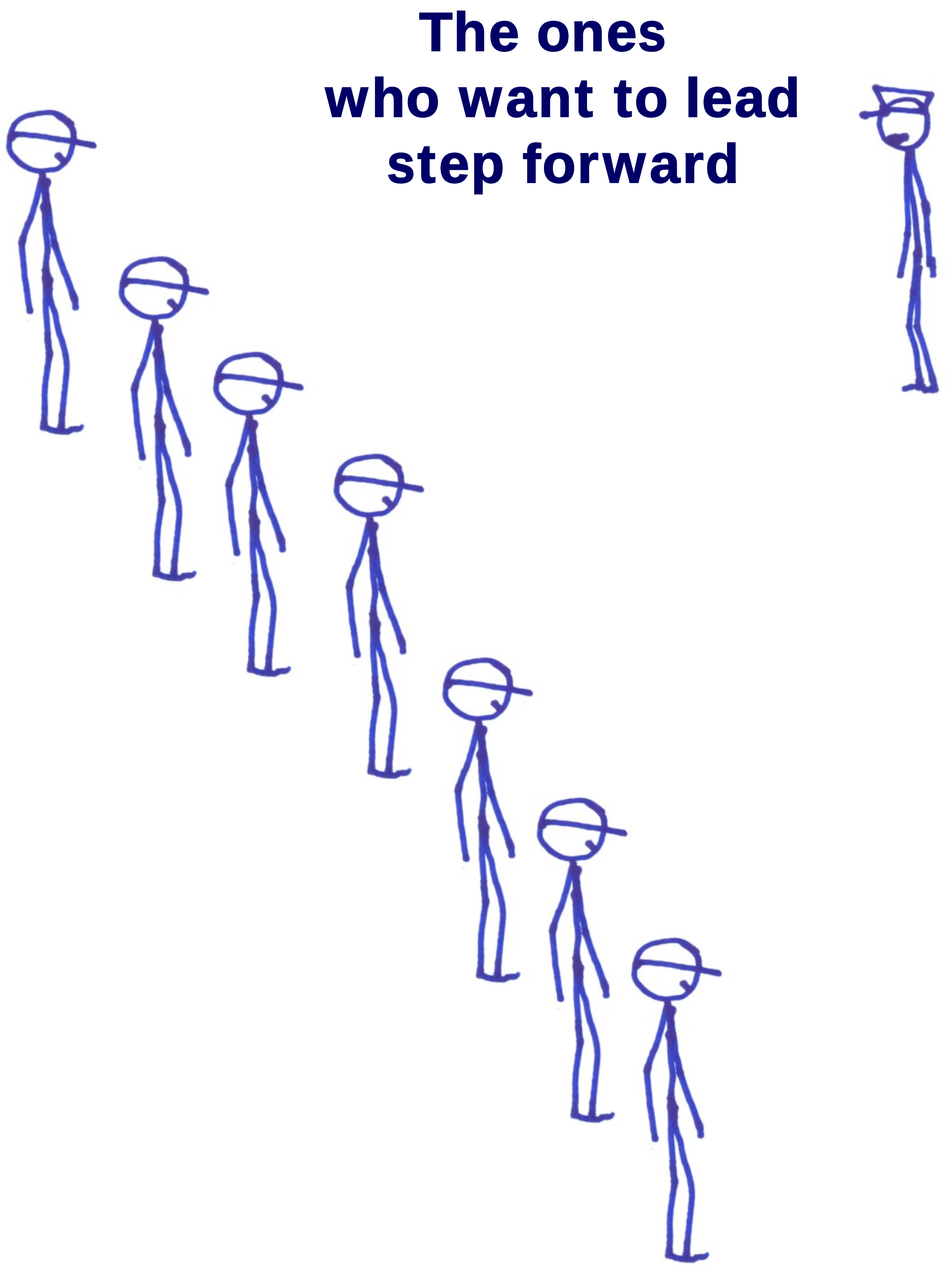
The secret of becoming a good leader-bird:

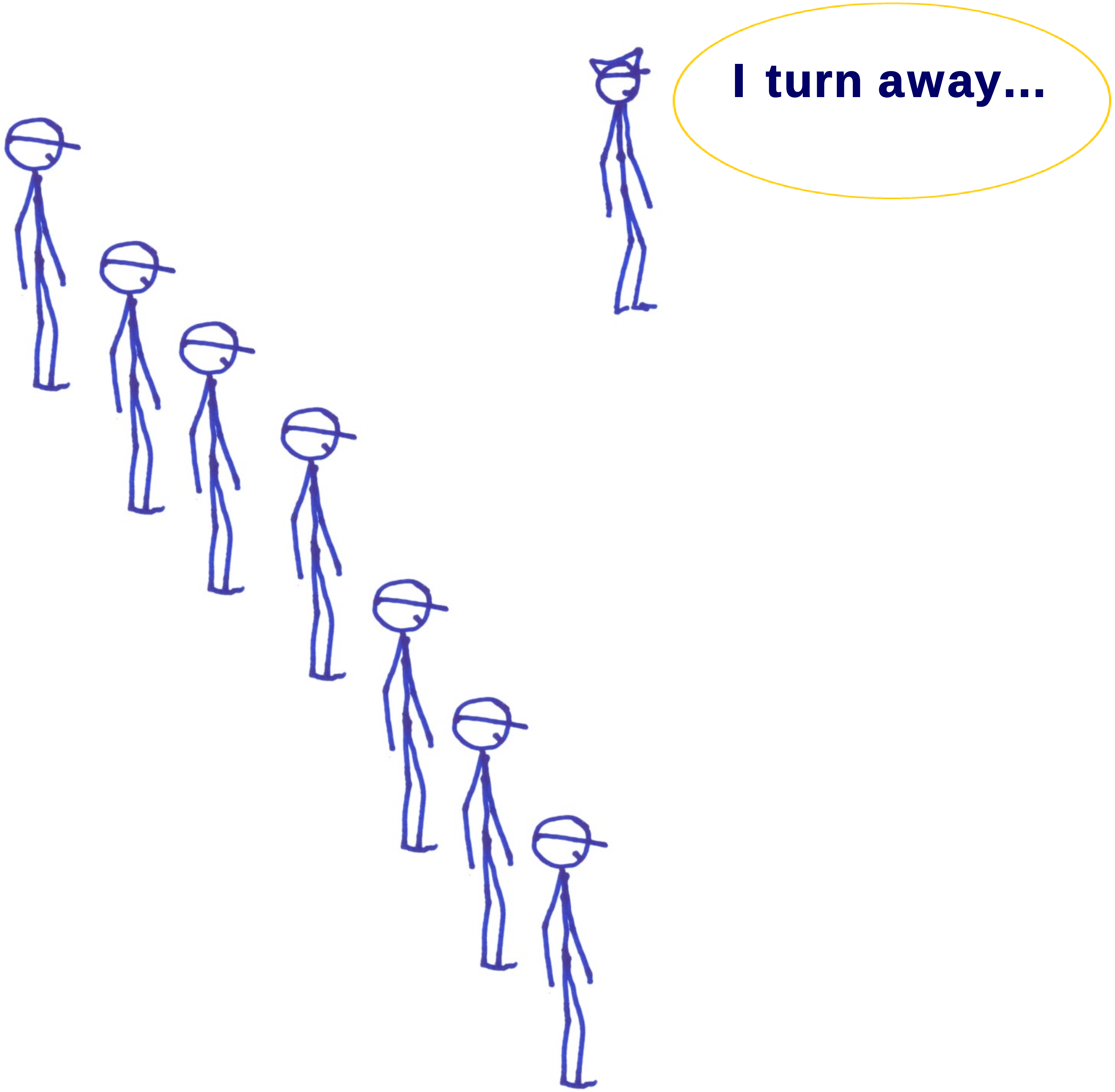
- If you fly faster, you tend to be in front

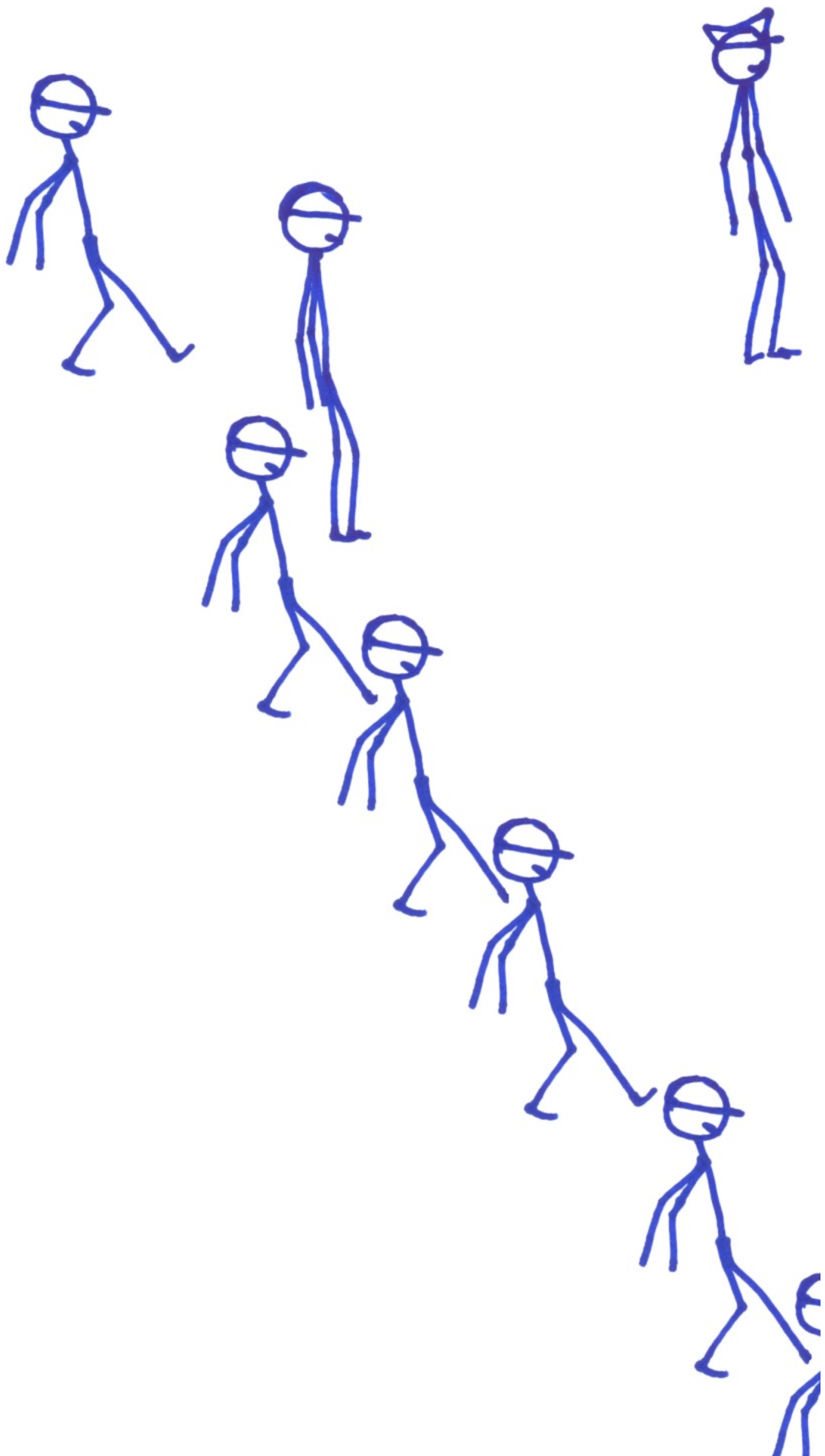
- To be in front „triggers” decision-making

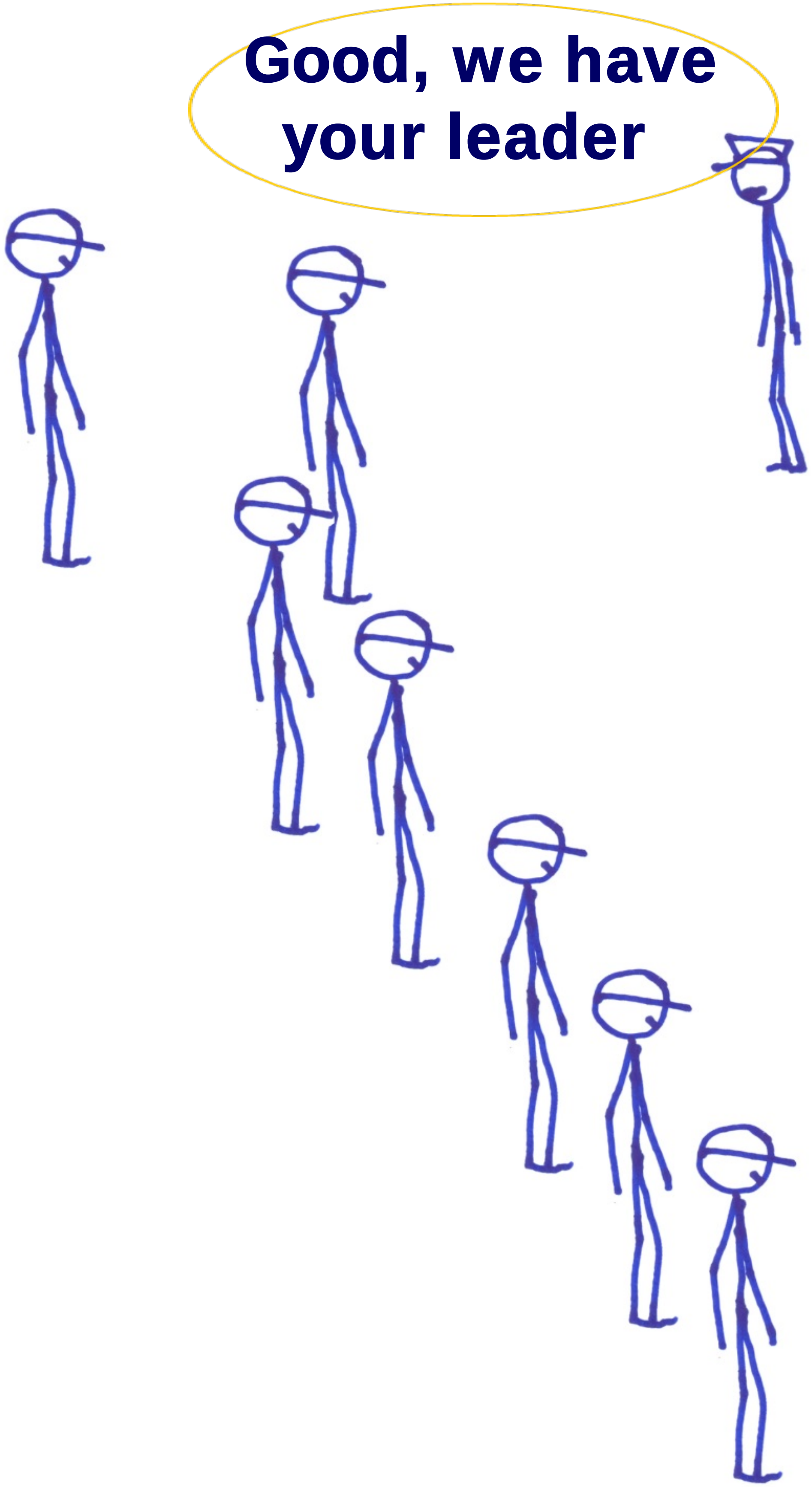
- Because you find yourself in situations in which you have to make decisions

- Makes you more experienced: a better leader

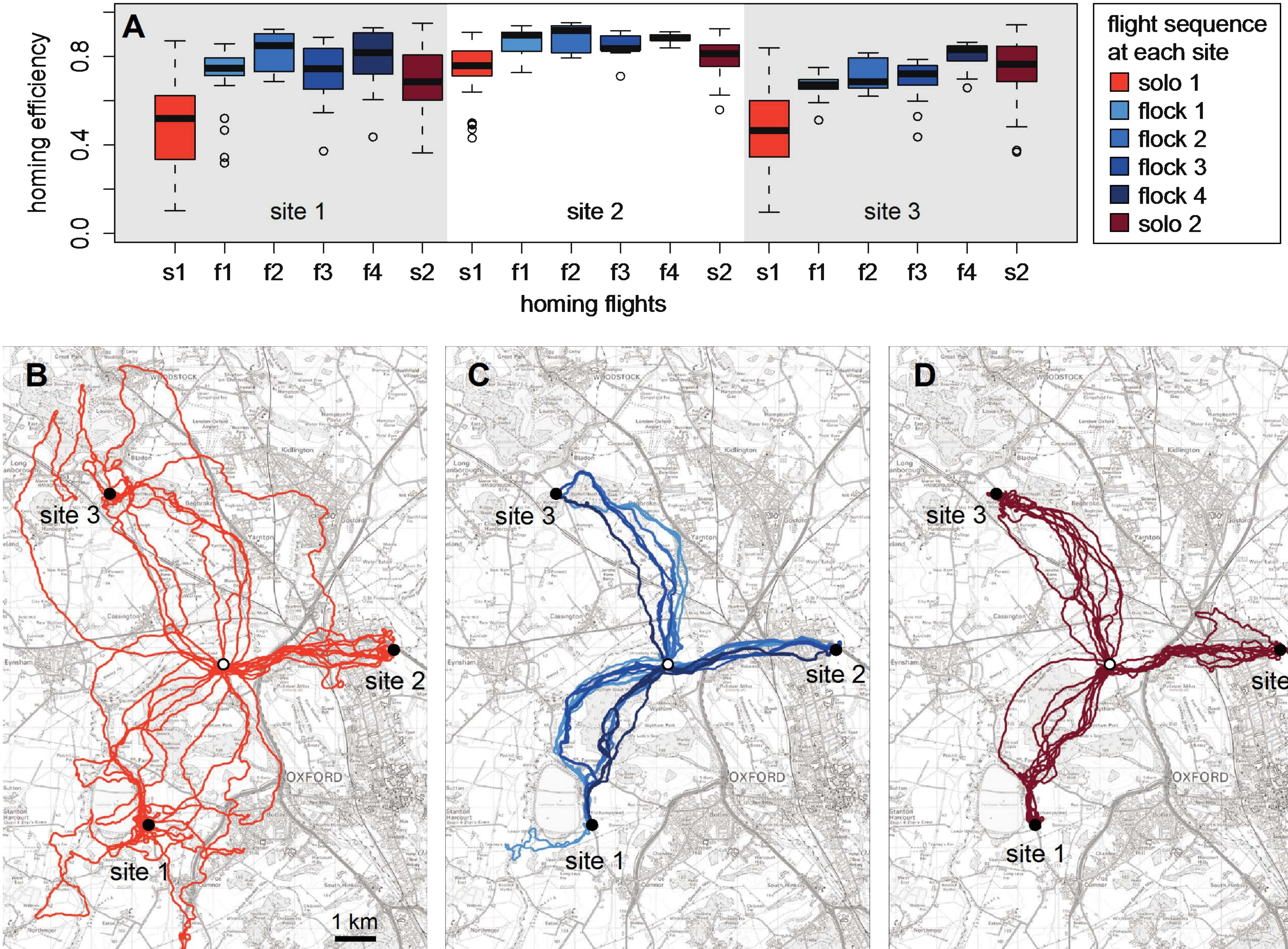

The ones
who want to lead
step forward


I turn away...



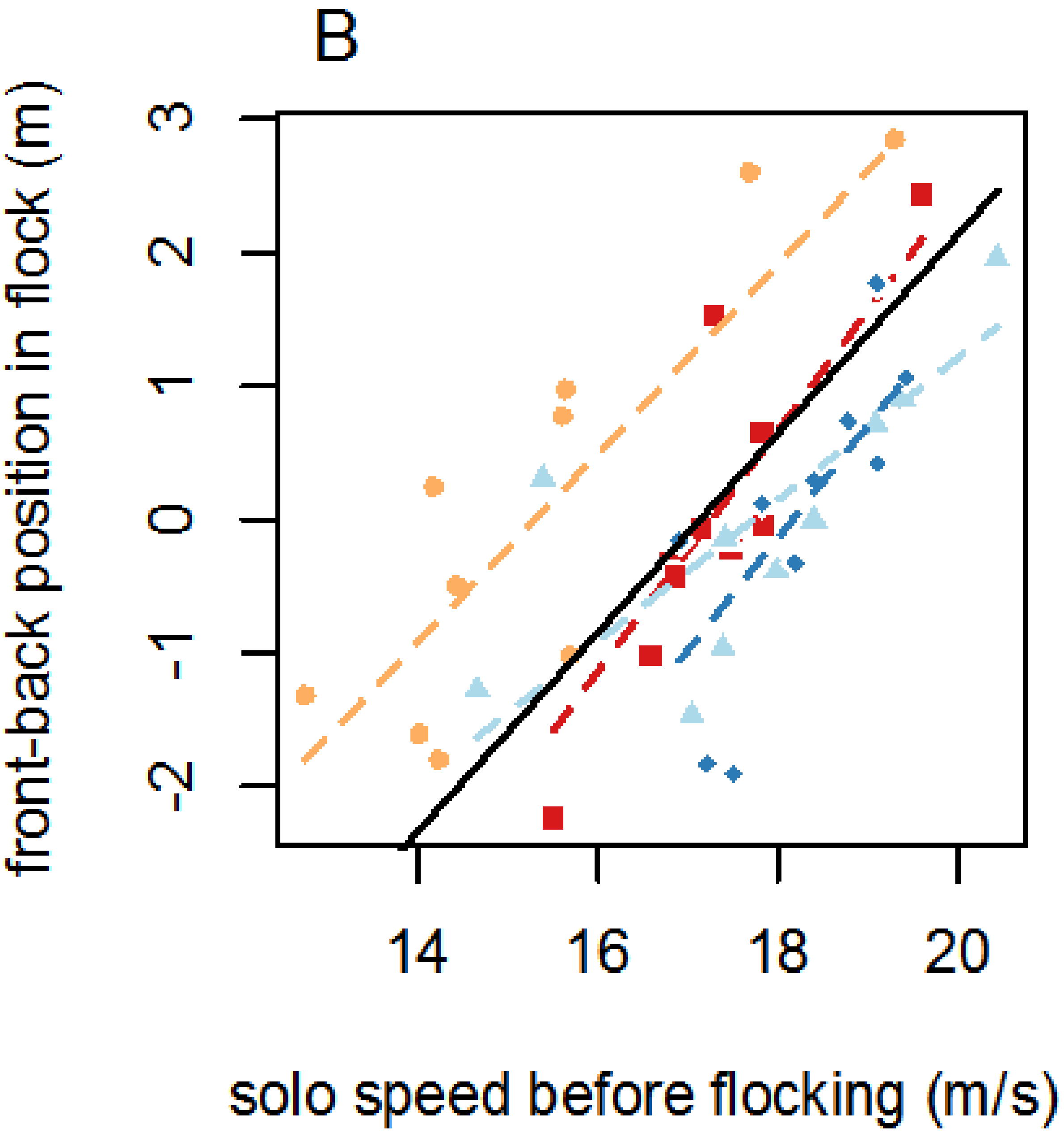

Good, we have your leader

Three sites, individual and group flights, 10 pigeons / flock

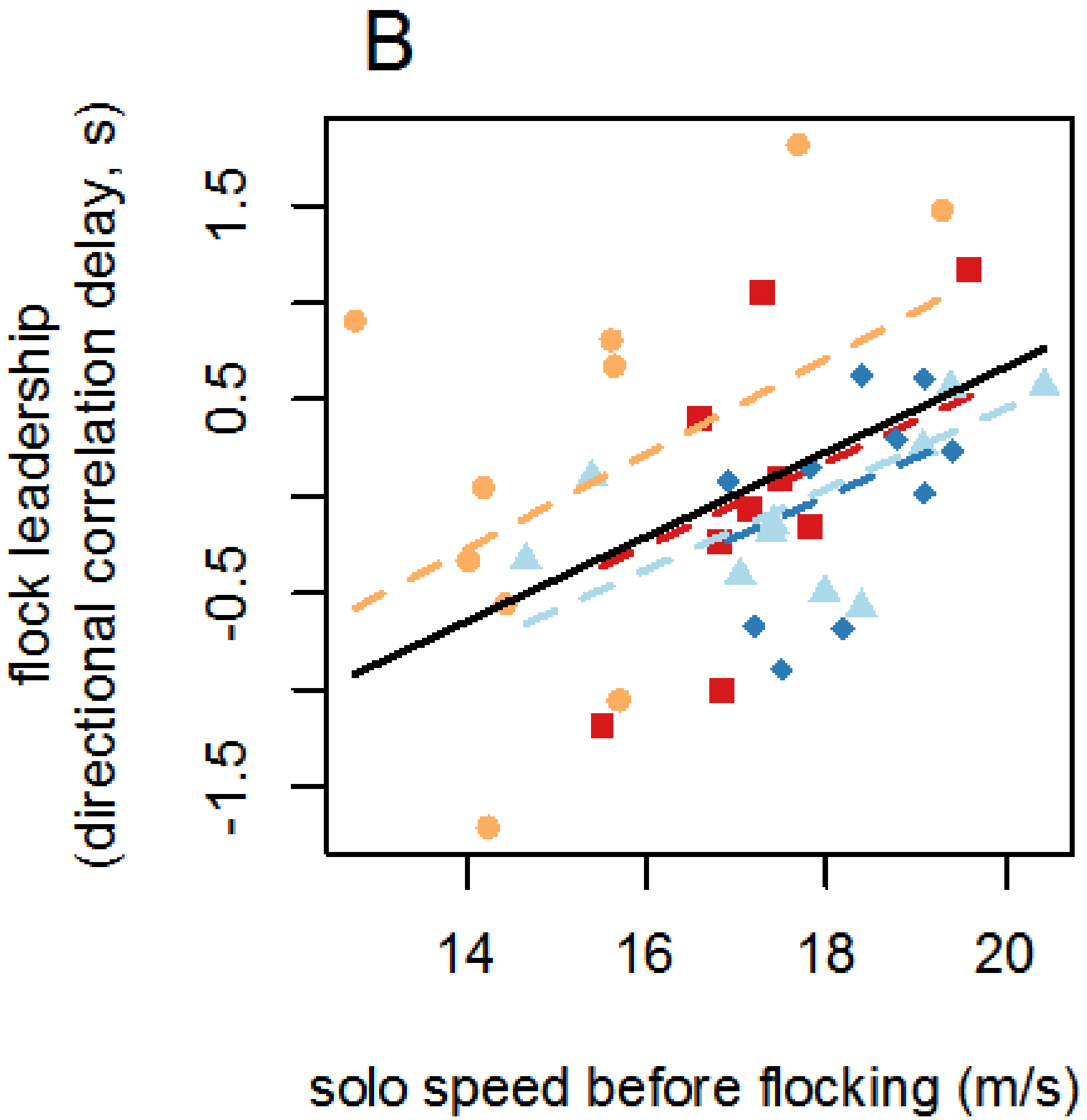


For taking these data one has to spend much time on the field. There are several more Google Earth(!!) pictures with our pigeons-carrying red car in them

Pigeon is in front if fast

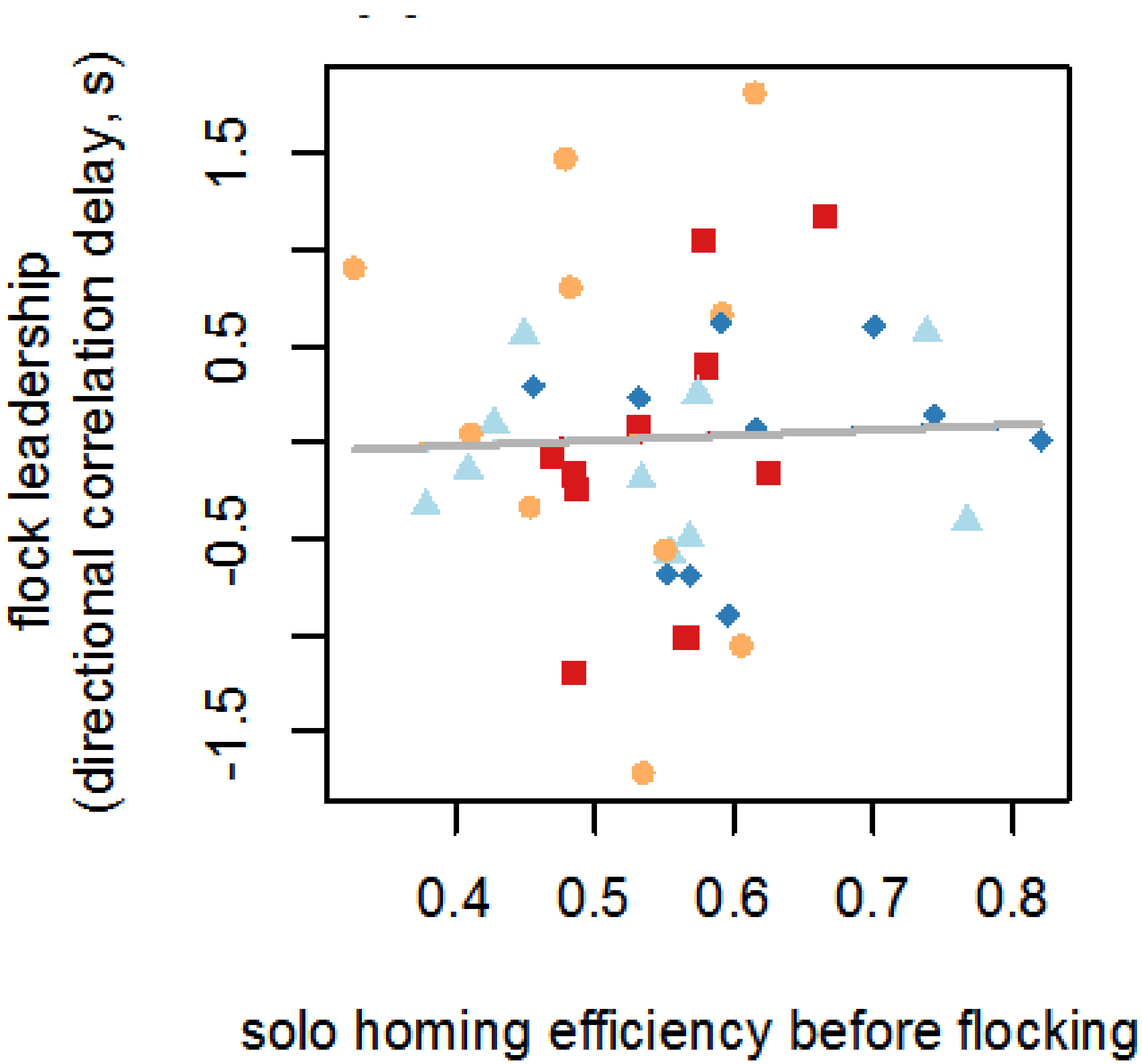


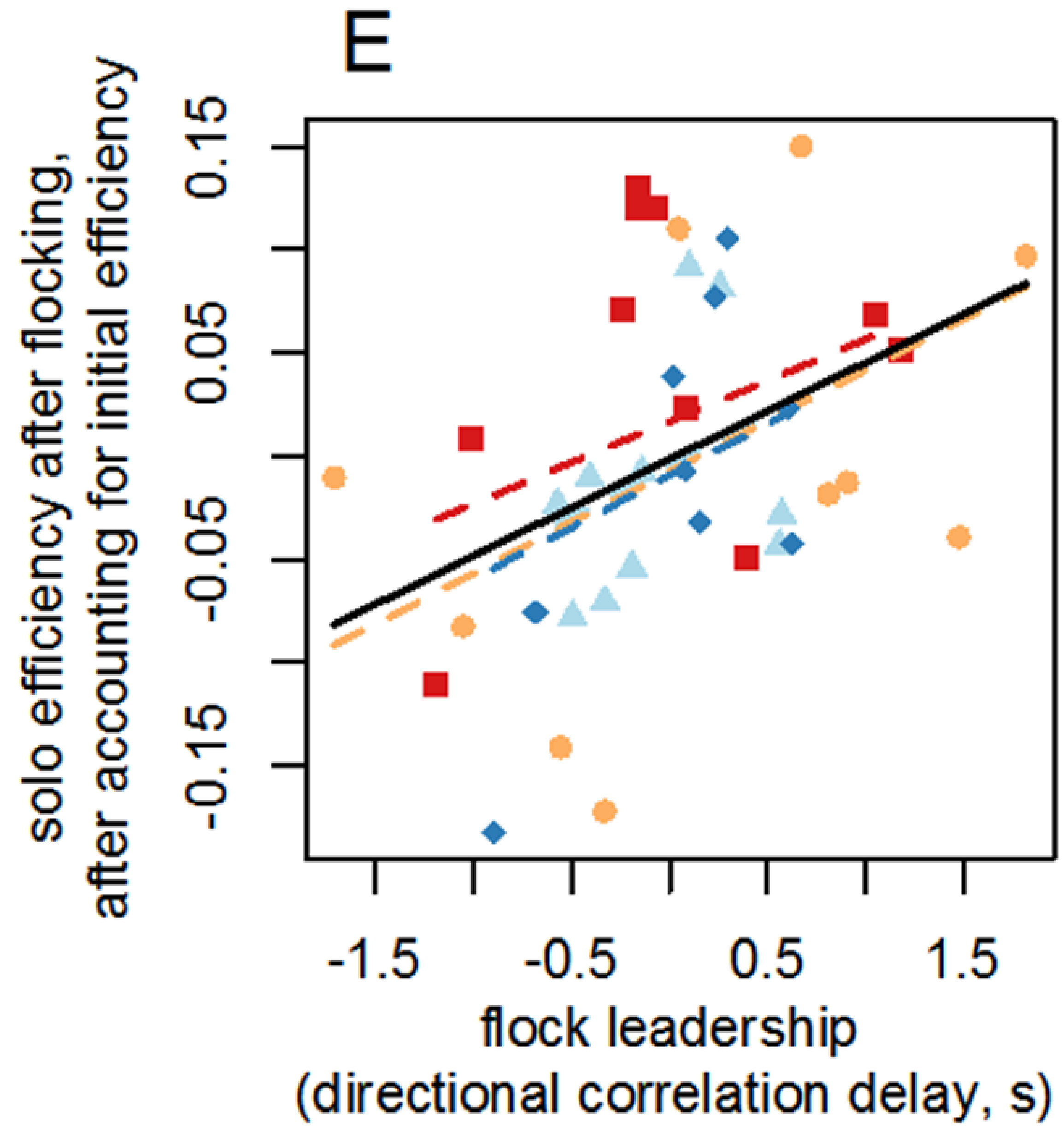
pigeon leads if fast



Pigeon does not lead if if not a good navigator

after flocking leaders become good navigators



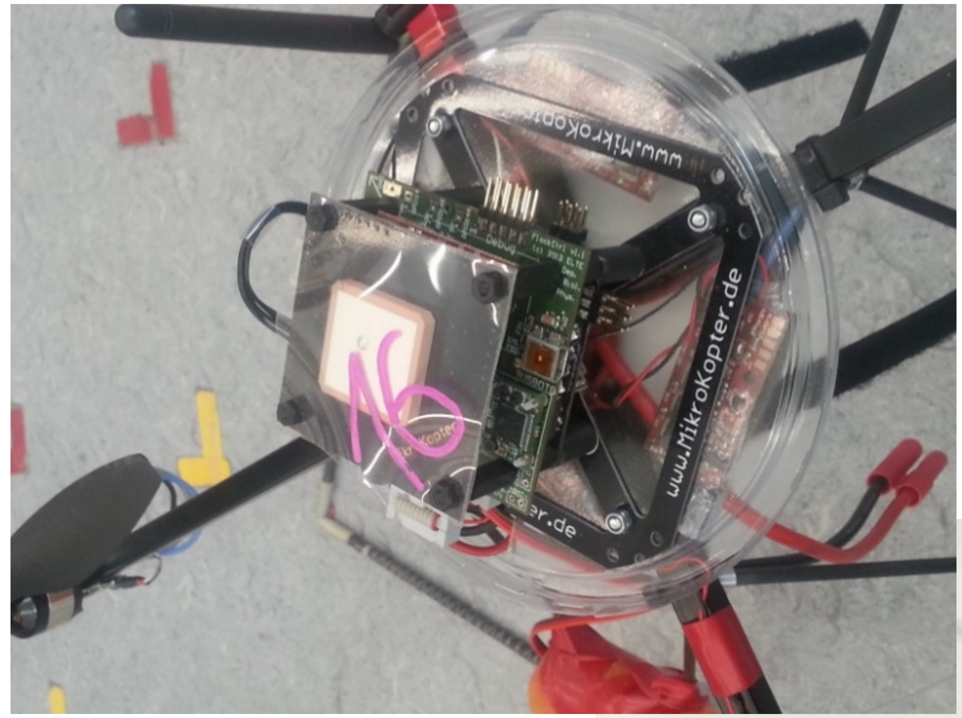


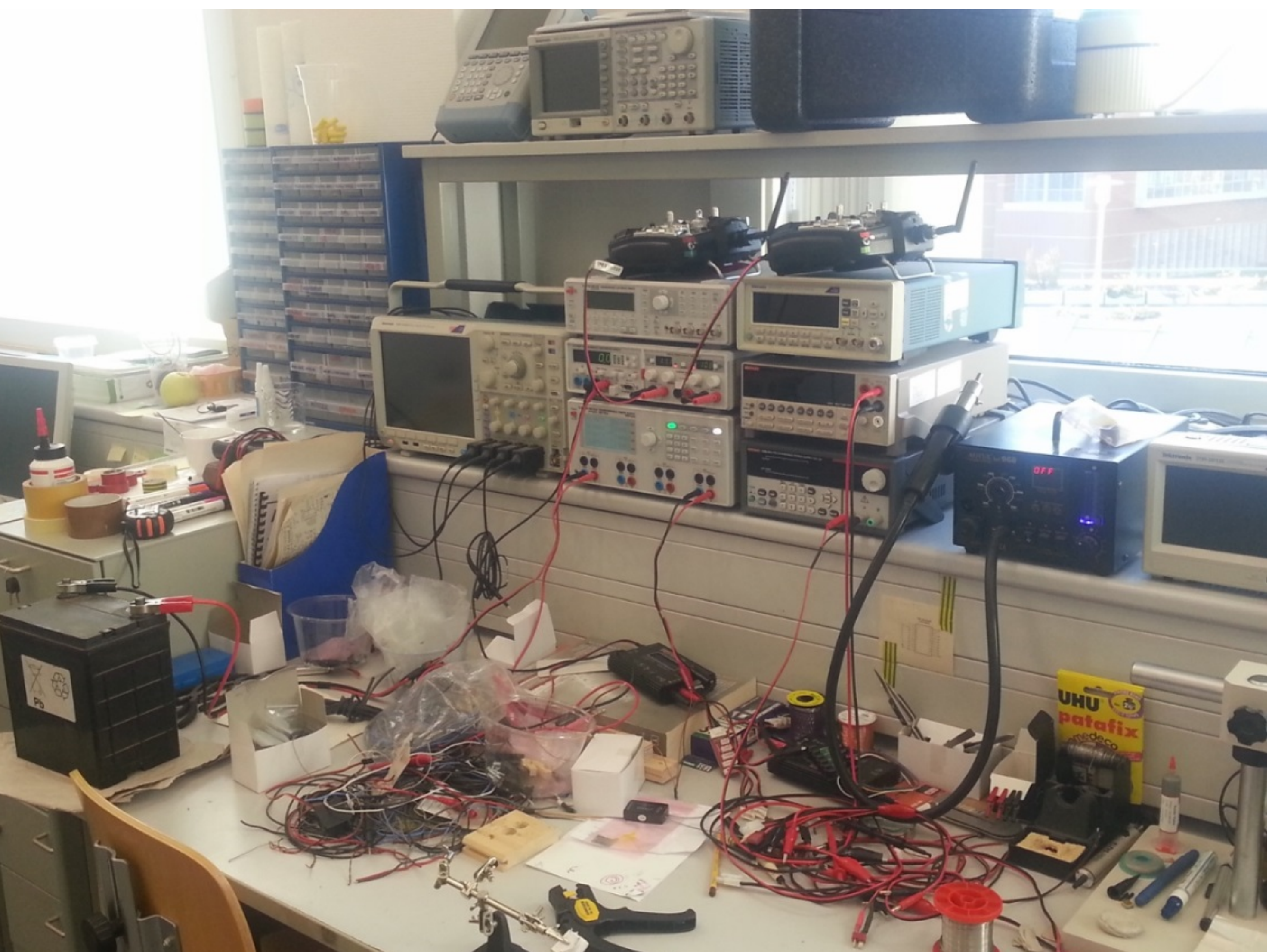
Technology and life are intimately related…

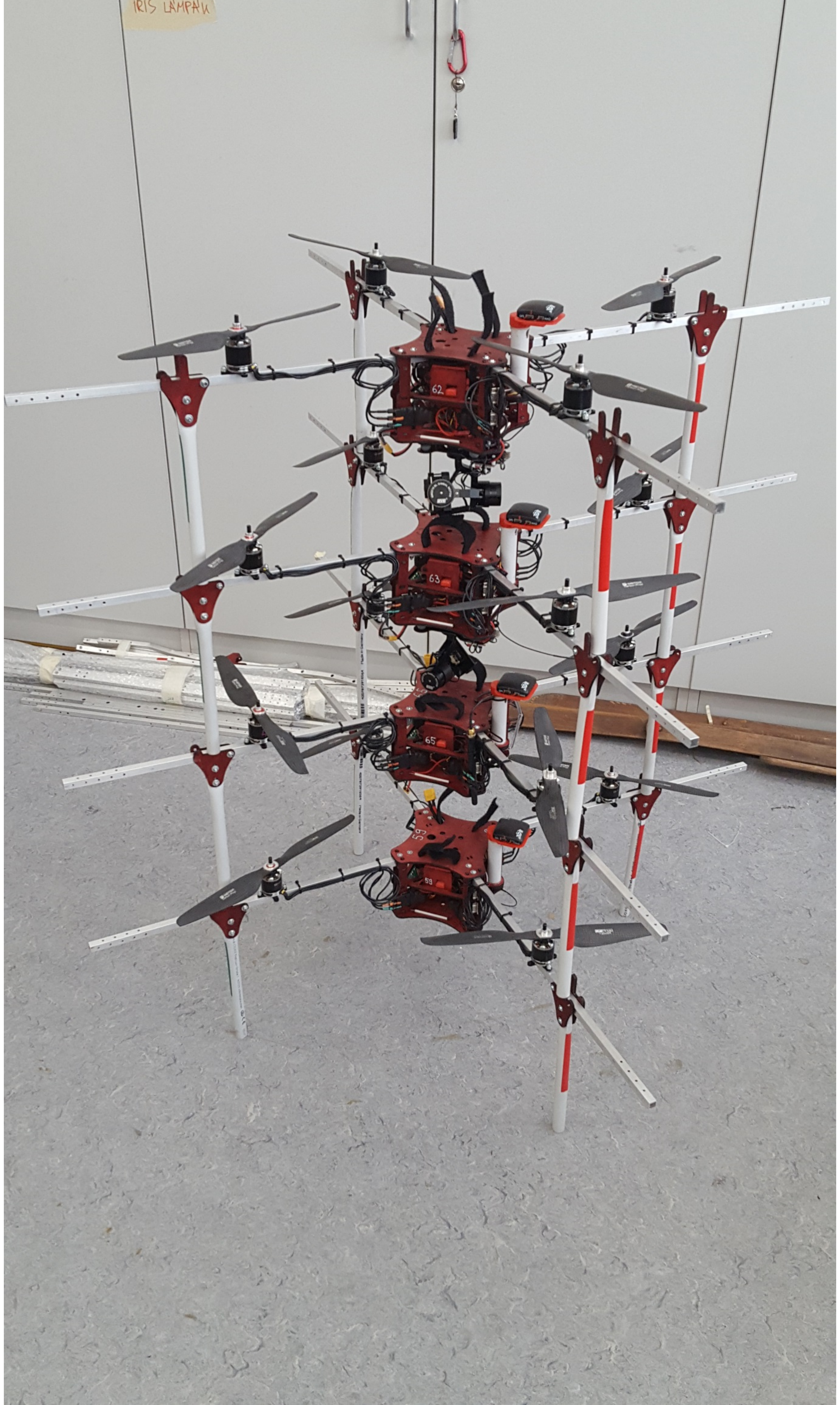
Robethology

The collective behaviour of autonomous quadcopters (NO central computer, communication only between the robots)

Our copters and lab



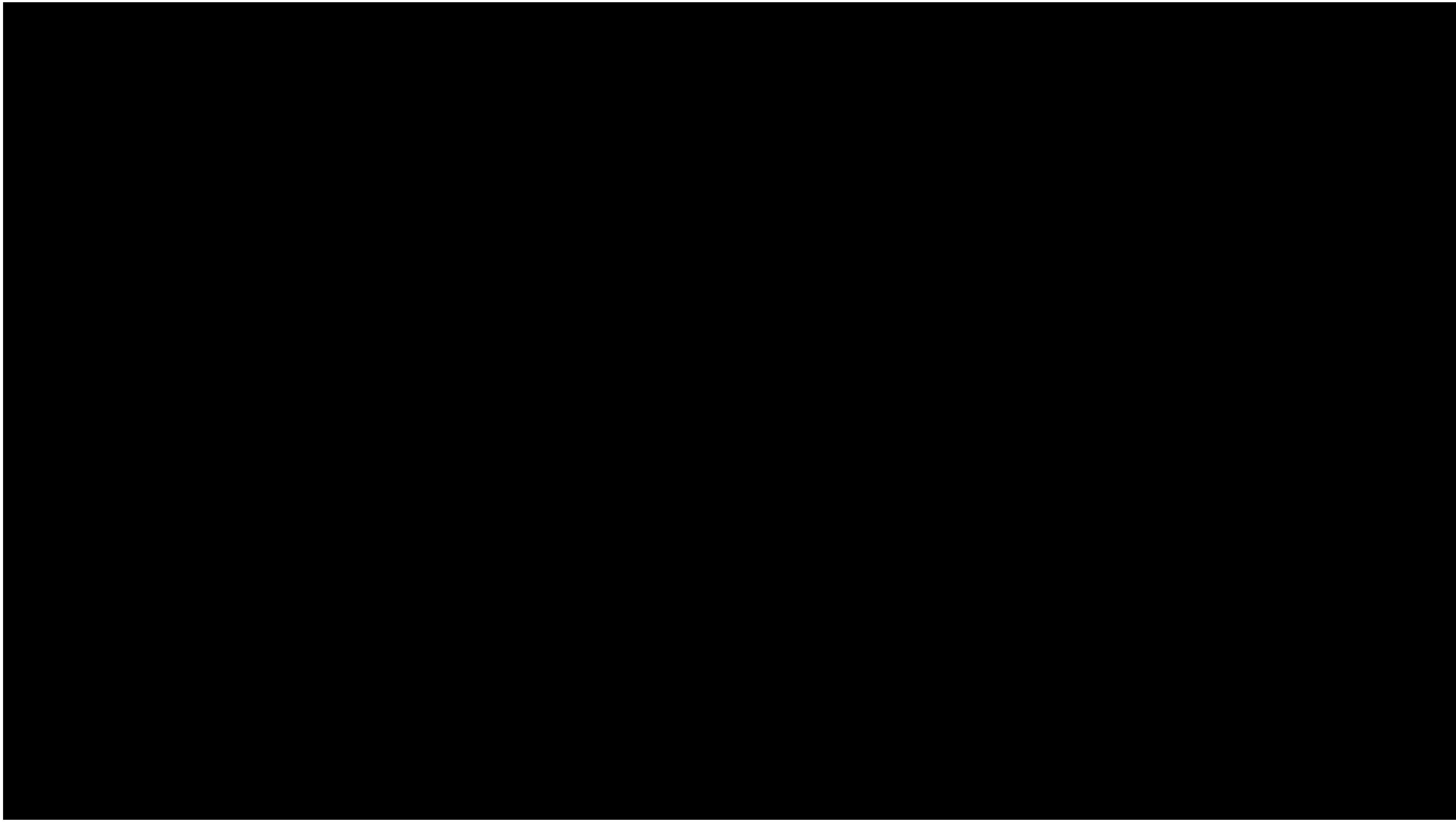





62
63

Interaction of a dancer with a flock of <u>autonomous</u> drones

(equations in the spirit of the chase and escape project)



Autonomuous chasing by drones
(Viceland, Canada, Teaser, Dec, 2016)

Many thanks are due to my collaborators

Principal collaborators:

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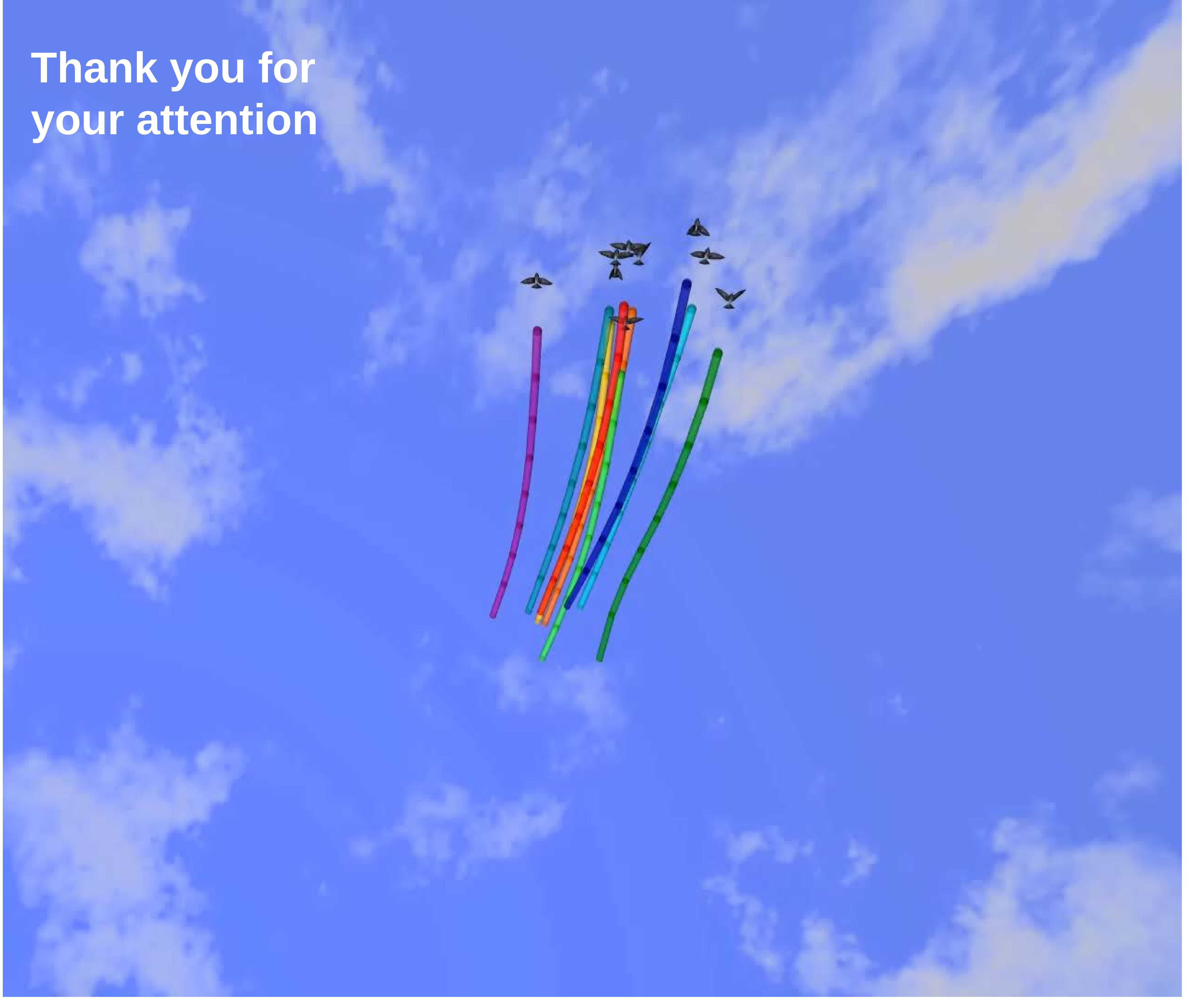
Kunal Bhattacharya
Dóra Biró (Oxford)
Máté Nagy
Tamás Nepusz
Benj Pettit (Oxford)
Gergely Somorjai
Gábor Vásárhelyi
+ many more

Support: Hungarian Science Foundation (OTKA)
EU FP6 Starflag project
EU ERC COLLMOT project


Thank you for
your attention

Thank you for your attention!

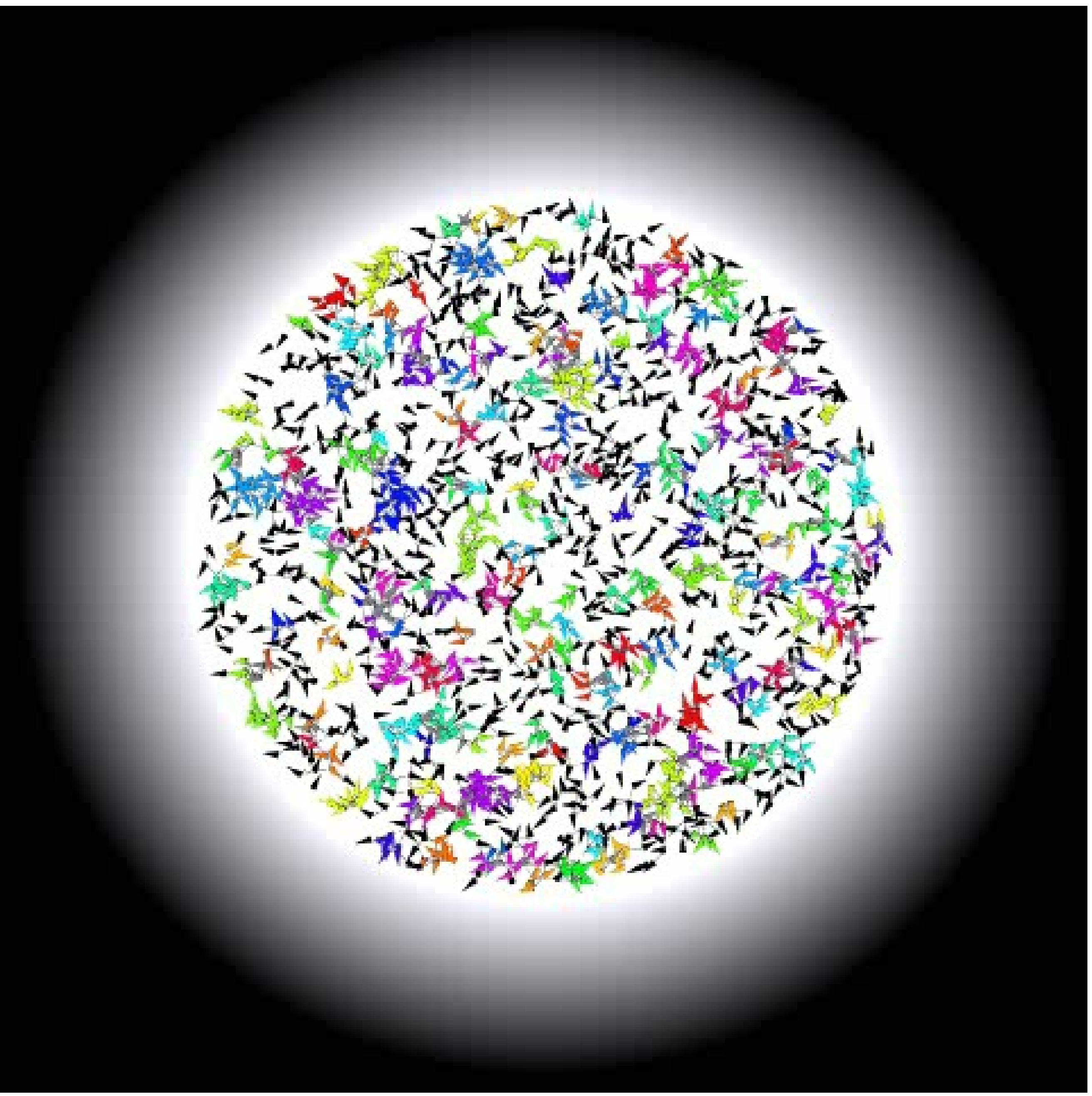
Dynamics of *k*-clique clusters

Two nodes belong to the same cluster if there is connected path of neighbouring *k*-cliques (overlapping cluster analysis of the underlying graph)

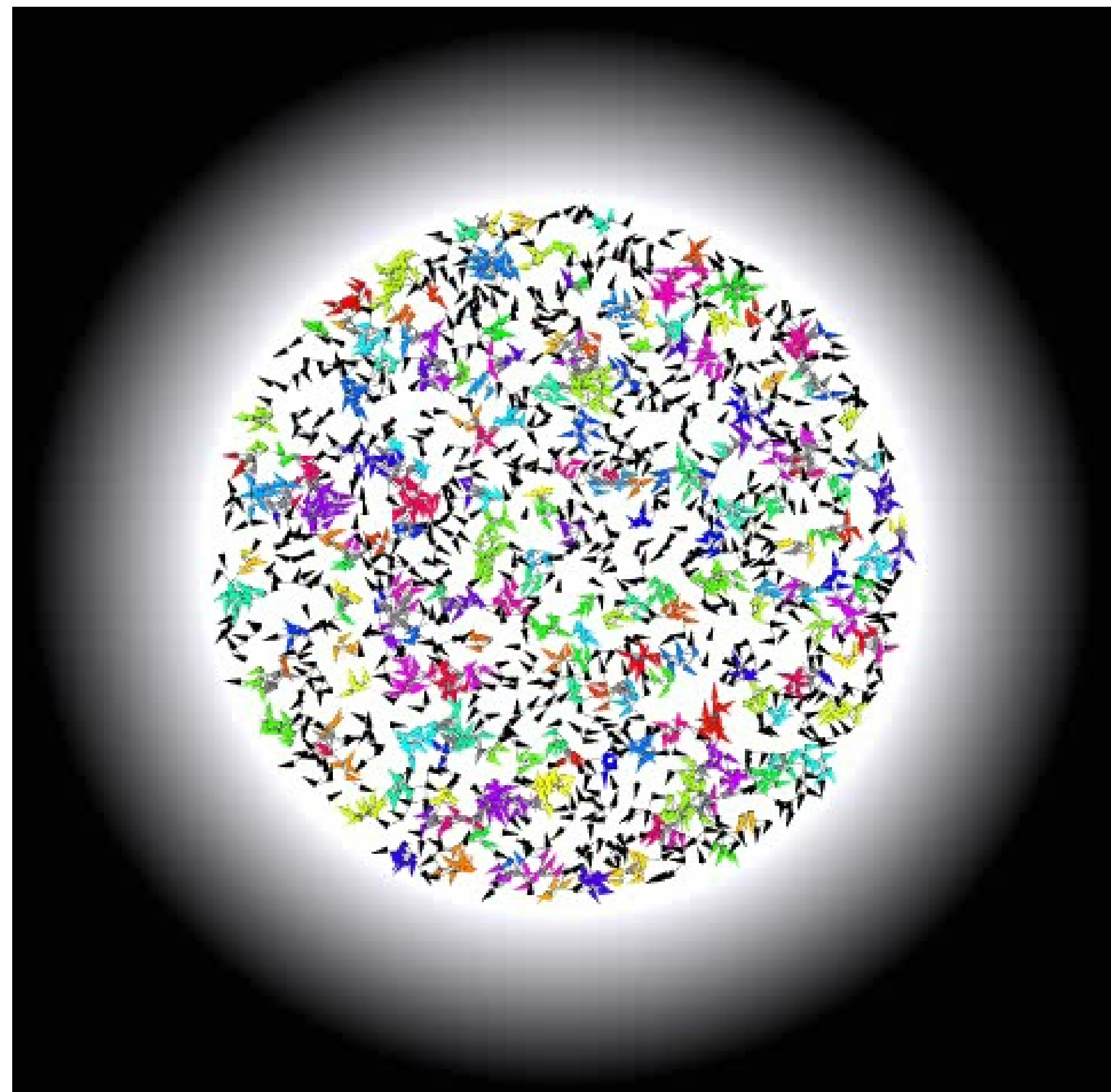
Here: $k = 4$

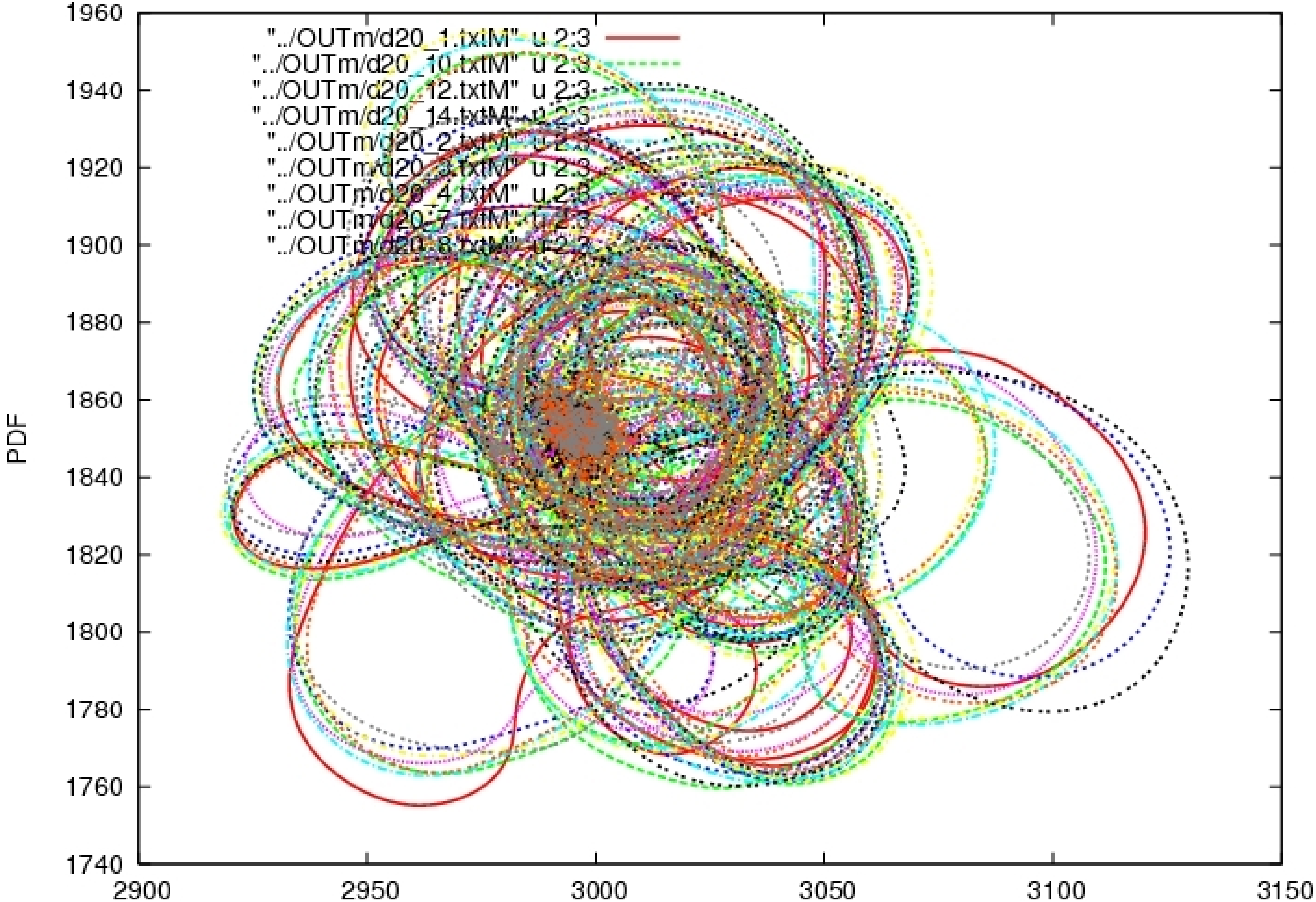
Method after Palla, Barabasi and T.V, *Nature*, 2007

□= 0.4



□= 0.3





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"../OUTm/d20_12.txtM" u 2:3
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"../OUTm/d20_2.txtM" u 2:3
"../OUTm/d20_3.txtM" u 2:3
"../OUTm/d20_4.txtM" u 2:3
"../OUTm/d20_7.txtM" u 2:3
"../OUTm/d20_8.txtM" u 2:3
PDF
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