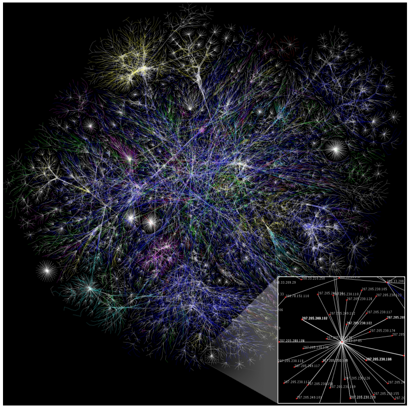


Modeling structure and function of complex networks and the brain

Remco van der Hofstad
(with Sándor Kolumbán)

ICMS Winterschool 'Complexity Science'

February 13-17, 2017

Technische Universiteit Eindhoven University of Technology

EURANDOM

NETWORKS

Organization

▷ Lecture 1:
Modeling Complex Networks

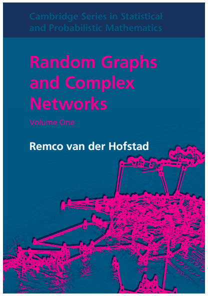
▷ Lecture 2:
Network Functionality

▷ Lecture 3:
Complex Networks and the Brain

(with Sándor Kolumbán)

Material

Book:



Random Graphs and Complex Networks

http://www.win.tue.nl/~rhofstad/NotesRGCN.html

http://www.win.tue.nl/~rhofstad/NotesRGCN.pdf

Aimed at graduate students in math, here take softer approach.

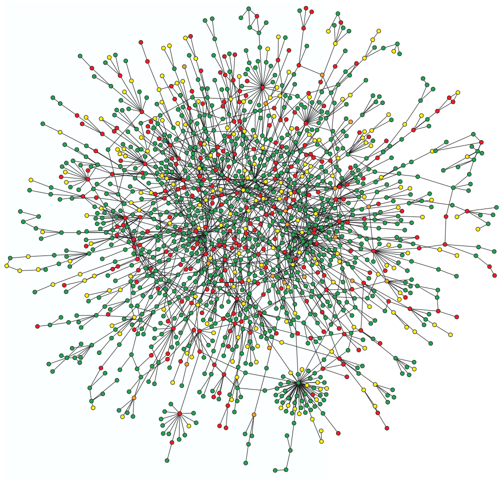
Yet, good source for informal explanations of random graphs for networks, and for basic results.
Non-mathematicians can ignore proofs...

Lecture 1:

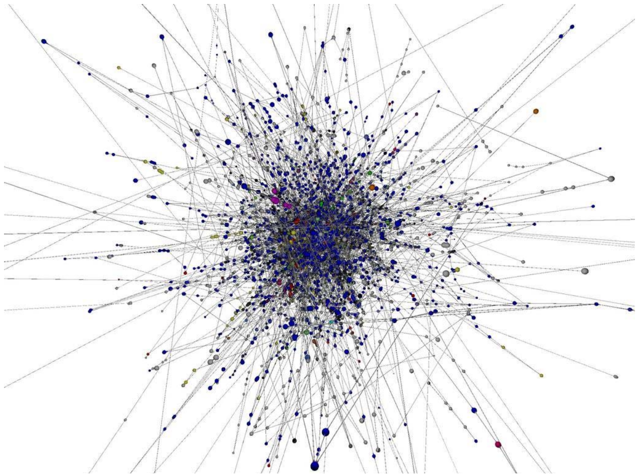
Modeling Complex Networks

- ▷ Complex Networks;
- ▷ Network statistics and Data;
- ▷ Models for complex networks.

Complex networks

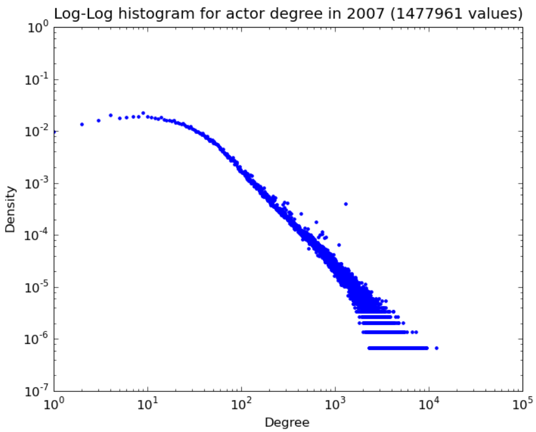


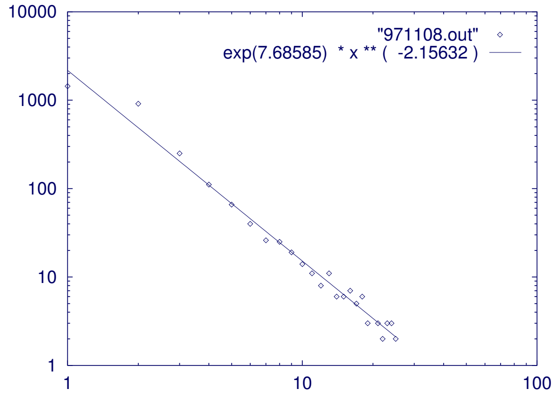
Yeast protein interaction network



Internet topology in 2001

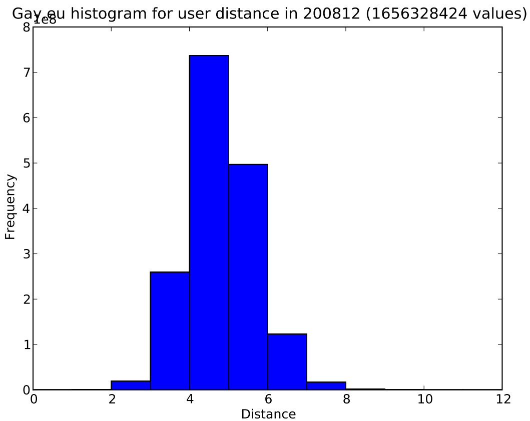
Scale-free paradigm

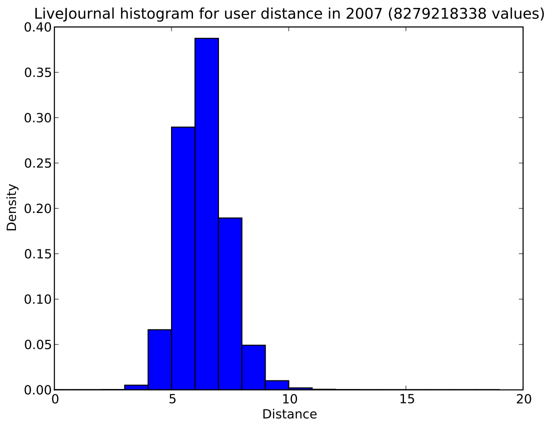




Loglog plot of degree sequences in Internet Movie Data Base (2007) and in the AS graph (FFF97)

Small-world paradigm





Distances in social networks `gay.eu` on December 2008 and `livejournal` in 2007.

Network statistics

▷ Clustering:

$$C = \frac{3 \times \text{ number of triangles}}{\text{number of connected triplets}}.$$

Proportion of friends that are friends of one another.

▷ Assortativity:

$$\rho = \frac{\frac{1}{|E_n|}\sum_{ij\in E_n} d_i d_j - \left(\frac{1}{|E_n|}\sum_{ij\in E_n} d_i\right)^2}{\frac{1}{|E_n|}\sum_{ij\in E_n} d_i^2 - \left(\frac{1}{|E_n|}\sum_{ij\in E_n} d_i\right)^2}.$$

Correlation between degrees at either end of edge.

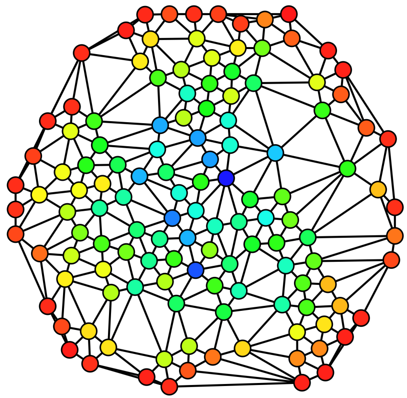
[Recent work vdH-Litvak (2013): flaws assortativity coefficient. Proposes rank correlations instead.]

Centrality measures

▷ Closeness centrality:
Measures to what extent vertex can reach others using few hops.
Vertices with low closeness centrality are central in network.

▷ Betweenness centrality:
Measures extent to which vertex connects various parts of network.

Betweenness large for bottlenecks.



▷ PageRank:
Measures extent to which vertex is visited by random walk.
Used in Google to rank importance in web pages.

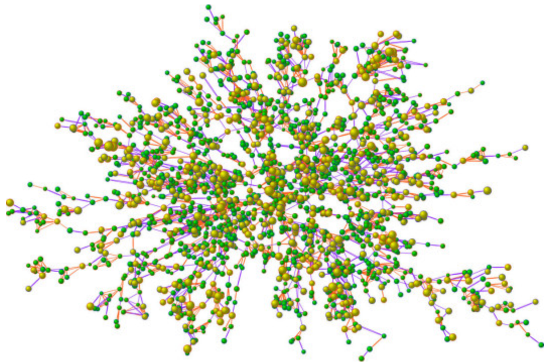
Modeling networks

Use random graphs to model uncertainty in formation

connections between elements.

▷ Static models:
Graph has fixed number of elements:

Configuration model



▷ Dynamic models:
Graph has evolving number of elements:

Preferential attachment model

Many models!

Universality??

Erdős-Rényi

Vertex set $[n] := \{1, 2, \ldots, n\}$.

Erdős-Rényi random graph is random subgraph of complete graph on $[n]$ where each of $\binom{n}{2}$ edges is occupied with probab. p.

Simplest imaginable model of a random graph.

$\triangleright$ Attracted tremendous attention since introduction 1959, mainly in combinatorics community.

Probabilistic method (Erdős et al).

$\triangleright$ Egalitarian: Every vertex has equal connection probabilities. Misses hub-like structure of real networks.

Inhomogeneous versions have been suggested and investigated.

Null model 1

Many adaptations, including the original Erdős-Rényi random graph, where a fixed number m of edges is chosen uniformly at random without replacement.

Models are closely related when taking

$$p \approx 2m/(n(n-1)).$$

Random graph with fixed number of edges is uniform random graph with that number of edges:

Null model.

Yields bench mark to compare real-world networks to having same number of edges.

$\triangleright$ Directed ERRG: One can also study directed versions.

Configuration model

$\triangleright$ Invented by Bollobás (1980)

to study number of graphs with given degree sequence.

Inspired by Bender+Canfield (1978)

Giant component: Molloy, Reed (1995)

Popularized by Newman, Strogatz, Watts (2001).

> $\triangleright$ n number of vertices;
> $\triangleright$ $\boldsymbol{d} = (d_1, d_2, \ldots, d_n)$ sequence of degrees is given.

Often take $(d_i)_{i\in[n]}$ to be sequence of independent and identically distributed (i.i.d.) random variables with certain distribution.

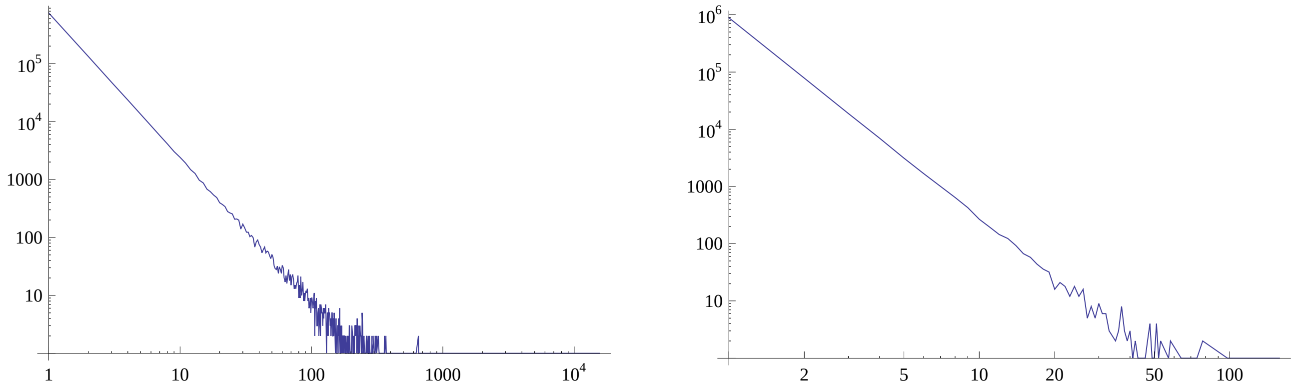
$\triangleright$ Special attention to power-law degrees, i.e., for $\tau > 1$ and c_τ

$$\mathbb{P}(d_1 \geq k) \approx c_\tau k^{-\tau+1}.$$

Power-laws CM

▷ Special attention to power-law degrees, i.e., for $\tau > 1$ and c_τ

$$\mathbb{P}(d_1 \geq k) = c_\tau k^{-\tau+1}(1 + o(1)).$$



Loglog plot of degree sequence CM with i.i.d. degrees $n = 1,000,000$ and $\tau = 2.5$ and $\tau = 3.5,$ respectively.

Graph construction

$\triangleright$ Assign d_j half-edges to vertex j. Assume total degree

$$\ell_n = \sum_{i \in [n]} d_i$$

is even.

$\triangleright$ Pair half-edges to create edges as follows:
Number half-edges from 1 to ℓ_n in any order.
First connect first half-edge at random with one of other $\ell_n - 1$ half-edges.

$\triangleright$ Continue with second half-edge (when not connected to first) and so on, until all half-edges are connected.

$\triangleright$ Resulting graph is denoted by $\mathrm{CM}_n(\boldsymbol{d})$.

Null model 2

Configuration model with fixed degrees and conditioned on simplicity yields uniform random graph with those degrees:

Null model.

Yields bench mark to compare real-world networks.

When degrees are not too heavy-tailed,

Probability simplicity uniformly positive.

$\triangleright$ Note: degrees in ERRG are close to Poisson, which does not fit well with many real-world networks.

$\triangleright$ Can also create uniform random graph with prescribed degrees by rewiring edges from any simple graph with those degrees. Is practical way to simulate graph. Problem: mixing time is unknown.

Graph distances in CM

H_n is graph distance between uniform pair of vertices in graph.

Theorem 1. (vdHHVM03). When $\nu = \mathbb{E}[D(D-1)]/\mathbb{E}[D] \in (1, \infty)$ and $\mathbb{E}[D_n^2] \to \mathbb{E}[D^2]$, conditionally on $H_n < \infty$,

$$\frac{H_n}{\log_\nu n} \xrightarrow{\mathbb{P}} 1.$$

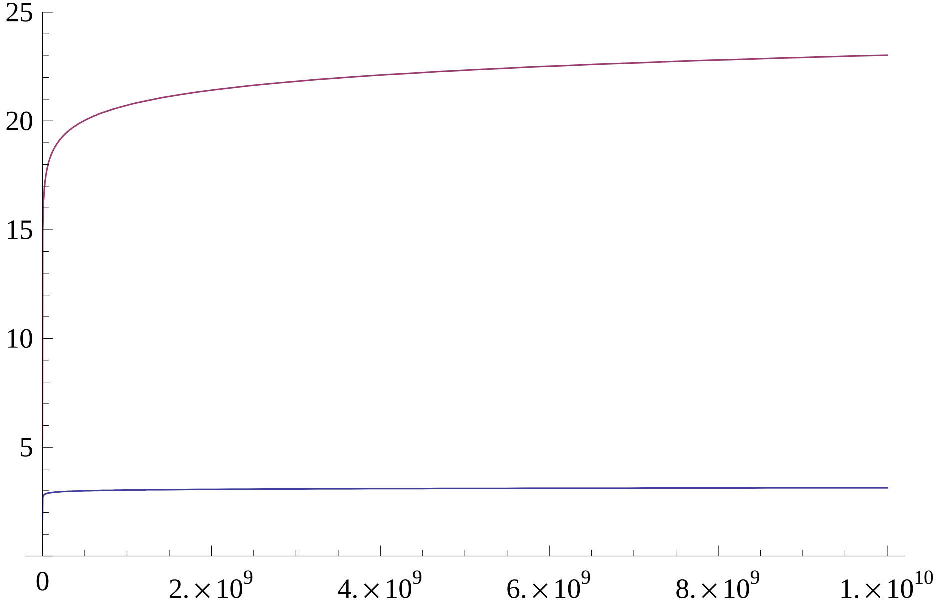
For i.i.d. degrees having power-law tails, fluctuations are bounded.

Theorem 2. (vdHHZ07, Norros+Reittu 04). When $\tau \in (2, 3)$, conditionally on $H_n < \infty$,

$$\frac{H_n}{\log \log n} \xrightarrow{\mathbb{P}} \frac{2}{|\log (\tau - 2)|}.$$

For i.i.d. degrees having power-law tails, fluctuations are bounded.

$x \mapsto \log\log x$ grows extremely slowly



Plot of $x \mapsto \log x$ and $x \mapsto \log\log x$.

Preferential attachment

Albert-Barabási (1999):
Emergence of scaling in random networks (Science).
26159 cit. (14-2-2017).
Bollobás, Riordan, Spencer, Tusnády (2001):
The degree sequence of a scale-free random graph process (RSA)

[In fact, Yule 25 and Simon 55 already introduced similar models.]

In preferential attachment models, network is growing in time, in such a way that new vertices are more likely to be connected to vertices that already have high degree.

Rich-get-richer model:

Preferential attachment

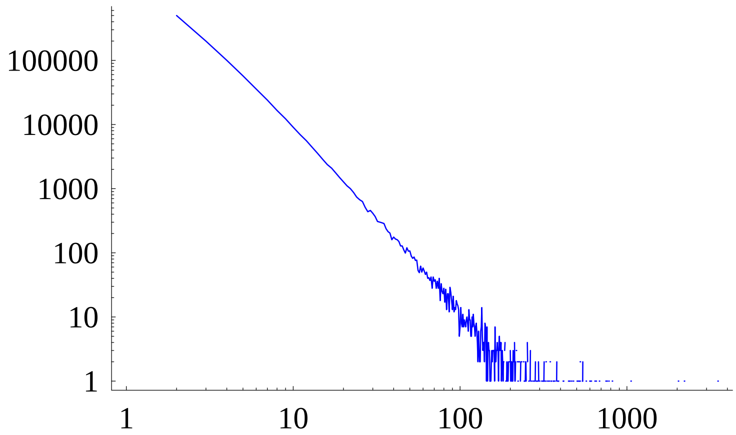
At time n, single vertex is added with m edges emanating from it. Probability that edge connects to $i^{\rm th}$ vertex is proportional to

$$D_i(n-1)+\delta,$$

where $D_i(n)$ is degree vertex i at time n, $\delta > -m$ is parameter.

Yields power-law degree sequence with exponent $\tau = 3+\delta/m > 2$.

BRST01 $\delta = 0$,
DvdEvdHH09,...



$m=2, \delta=0, \tau=3, n=10^6$

Distances PA models

Theorem 3 (Bol-Rio 04). For all $m \geq 2$ and $\tau = 3$,

$$H_n = \frac{\log n}{\log\log n}(1 + o_{\mathbb{P}}(1)).$$

Theorem 4 (Dommers-vdH-Hoo 10). For all $m \geq 2$ and $\tau \in (3, \infty)$,

$$H_n = \Theta(\log n).$$

Theorem 5 (Dommers-vdH-Hoo 10, DerMonMor 11). For all $m \geq 2$ and $\tau \in (2, 3)$,

$$\frac{H_n}{\log\log n} \xrightarrow{\mathbb{P}} \frac{4}{|\log(\tau - 2)|}.$$

Network modeling mayhem

Models:

- ▷ Configuration Model
- ▷ Inhomogeneous Random Graphs
- ▷ Preferential Attachment Model

What is bad about these models?

- ▷ Low clustering and few short cycles (unlike social networks);
- ▷ No communities (unlike collaboration networks and WWW);
- ▷ No attributes (geometry, gender,...);

Models are caricature of reality!

Conclusions

▷ Networks useful way to view real-world phenomena:

centrality and clustering.

▷ Unexpected commonality networks:

scale free and small worlds.

▷ Random graph models:

$$\boxed{\text{Used to explain properties of real-world networks and as benchmark.}}$$

Lecture 2:

Network Functionality

- ▷ Competition processes;
- ▷ Ising model and consensus reaching.

Competition

▷ Viral marketing aims to use social networks so as to excelerate adoption of novel products.

▷ Observation: Often one product takes almost complete market. Not always product of best quality:

Why?

▷ **Aim:** Explain this phenomenon, and relate it to network structure as well as spreading dynamics.

Competition: setting

▷ Model social network as random graph:

configuration model.

▷ Model dynamics as competing rumors spreading through network:

vertices, once occupied by certain type, try to occupy their neighbors at (possibly) random and i.i.d. times:

Fastest type corresponds to best product.

Questions:

▷ Who wins?
▷ How much does loosing type get?
▷ How do results depend on network topology?
▷ How do results depend on spreading dynamics?

Markovian spreading

Theorem 6. [Deijfen-vdH (2013)] Fix $\tau \in (2,3)$.
Consider competition model, where types compete for territory at fixed, but possibly unequal rates. Then, each of types wins majority vertices with positive probability:

$$\frac{N_1}{n} \xrightarrow{d} I \in \{0,1\}.$$

Number of vertices for losing type converges in distribution:

$$N_{\mathrm{los}}(n) \xrightarrow{d} N_{\mathrm{los}} \in \mathbb{N}.$$

The winner takes it all, the loser's standing small...

▷ Who wins is determined by location of starting point types:

Location, location, location!

Deterministic spreading

Theorem 7. [Baroni-vdH-Komjáthy (2014)] Fix $\tau \in (2,3)$.
Consider competition model, where types compete for territory with deterministic traversal times. Without loss of generality, assume that traversal time type 1 is 1, and of type 2 is $\lambda \geq 1$.

Fastest types wins majority vertices, i.e., for $\lambda > 1$,

$$\frac{N_1(n)}{n} \xrightarrow{\mathbb{P}} 1.$$

Number of vertices for losing type 2 satisfies that there exists random variable Z s.t.

$$\frac{\log(N_2(n))}{(\log n)^{2/(\lambda+1)} C_n} \xrightarrow{d} Z.$$

▷ Here, C_n is some random oscillatory sequence.

Deterministic spreading

Theorem 8. [vdH-Komjáthy (2014)] Fix $\tau \in (2,3)$.
Consider competition model, where types compete for territory with deterministic equal traversal times.

▷ When starting locations of types are sufficiently different,

$$\frac{N_1(n)}{n} \xrightarrow{d} I \in \{0,1\},$$

and number of vertices for losing type satisfies

$$\frac{\log(N_{\mathrm{los}}(n))}{C_n \log n} \xrightarrow{d} Z,$$

where $C_n \in (0,1)$ whp.
▷ When starting locations are sufficiently similar, **coexistence** occurs, i.e., there exist $0 < c_1, c_2 < 1$ s.t. whp

$$\frac{N_1(n)}{n}, \frac{N_2(n)}{n} \in (c_1, c_2).$$

Insight: local analysis

▷ Competition is to large extent determined by local behavior around starting points, including

discovery process early on in game.

▷ Configuration model is

locally tree-like.

Means that local neighborhood around a vertex is close to tree, i.e., no cycles. Make life much **easier.**

□ Thus, need to understand process on tree in detail.

Neighborhoods CM

$\triangleright$ Important ingredient in proof is description local neighborhood of uniform vertex $U_1 \in [n]$. Its degree has distribution $D_{U_1} \stackrel{d}{=} D$.

$\triangleright$ Take any of D_{U_1} neighbors a of U_1. Law of number of forward neighbors of a, i.e., $B_a = D_a - 1$, is approximately

$$\mathbb{P}(B_a = k) \approx \frac{(k+1)}{\sum_{i\in[n]} d_i} \sum_{i\in[n]} \mathbb{1}_{\{d_i=k+1\}} \xrightarrow{\mathbb{P}} \frac{(k+1)}{\mathbb{E}[D]}\mathbb{P}(D = k+1).$$

Equals size-biased version of D minus 1. Denote this by $D^\star - 1$.

Local tree-structure CM

$\triangleright$ Forward neighbors of neighbors of U_1 are close to i.i.d. Also forward neighbors of forward neighbors have asymptotically same distribution...

$\triangleright$ Conclusion: Neighborhood looks like branching process with offspring distribution $D^\star - 1$ (except for root, which has offspring D.)

$\triangleright$ $\tau \in (2,3):$ Infinite-mean BP, which has double exponential growth of generation sizes:

$$(\tau-2)^k \log(Z_k \vee 1) \xrightarrow{a.s.} Y \in (0,\infty).$$

Graph distances CM

H_n is graph distance between uniform pair of vertices in $\mathrm{CM}_n(\boldsymbol{d})$.

Theorem 2. [vdHHZ07, Norros-Reittu 04]. Fix $\tau \in (2,3)$. Then,

$$\frac{H_n}{\log\log n} \xrightarrow{\mathbb{P}} \frac{2}{|\log(\tau-2)|},$$

and fluctuations are tight, but do not converge in distribution.

$\triangleright$ In absence of competition, it takes each of types about $\frac{\log\log n}{|\log(\tau-2)|}$ steps to reach vertex of maximal degree.

$\triangleright$ Type that reaches vertices of highest degrees (=hubs) first wins. When $\lambda > 1$, fastest type wins whp.

Deterministic dynamics

$\triangleright$ With $Z_k^{(i)}$ number of vertices reachable in precisely k steps by type i, in absence of competition, as long as k is 'small' wrt $\frac{\log\log n}{|\log(\tau-2)|}$,

$$(\tau-2)^k \log Z_k^{(i)} \xrightarrow{d} Y^{(i)}.$$

$\triangleright$ When fastest type has occupied hubs, i.e., vertices of highest degrees, it occupies all vertices of smaller and smaller degrees.

$\triangleright$ Number of vertices of losing kind can be computed by investigating at what degrees types encounter each other first.

$\triangleright$ Coexistence occurs when both vertices find hubs at same time, which occurs when

$$Y^{(1)}/Y^{(2)} \in (\tau-2, 1/(\tau-2)).$$

Ising Model

Ising model is spin system, where vertices can be in two states $\{-1, 1\}$. Invented as model for magnetism. Think of 1 indicating one opinion, -1 as opposite opinion.

Ising model describes collection of opinions using Boltzman distribution: Let $G = (V, E)$ denote social network, and $\sigma = (\sigma_i)_{i \in V}$ as collection of opinions. Then, distribution of opinions equals

$$\mu_{\beta,B}(\sigma) = \frac{1}{Z(\beta, B)} \mathrm{e}^{-H(\sigma)},$$

where $H(\sigma)$ is Hamiltonian of configuration of opinions σ

$$H(\sigma) = -\beta \sum_{(x,y) \in E} \sigma_x \sigma_y - B \sum_{x \in V} \sigma_x.$$

$Z(\beta, B)$ is normalization constant. Here $\beta > 0$ determines preference for consensus, and $B \in \mathbb{R}$ determines preference opinion 1.

Ising model dynamics

At time $t \geq 0$, one randomly chosen person v changes his/her opinion σ_v to $1 - \sigma_v$ with probability

$$\min\left\{ \mathrm{e}^{-[H(\sigma^v) - H(\sigma)]}, 1 \right\},$$

where σ^v is obtained from σ by flipping status of v.
Measure $\mu_{\beta,B}$ is stationary distribution.

Finite systems do not have phase transitions. However, these may arise in the infinite-volume limit:

Phase transition:
Exists critical β_c. Below it stationary distribution is unique for $h = 0$.
Above it, two distinct stationary distributions (obtained from configurations where all spins agree):

Positive instantaneous magnetization.

Equilibrium Ising model

▷ Dynamics has preference for neighbors to align:

quest for order.

Opposing entropy effect:

many more configurations with many sign changes.

β is inverse temperature.

▷ Order wins when temperature low, alignment effect is high.

▷ Entropy wins when temperature high, alignment effect is not strong enough to enforce order.

Here, focus on

equilibrium behavior.

Key quantities

Magnetization:

$$M_n(\beta, B) = \left\langle \frac{1}{n} \sum_{i \in [n]} \sigma_i \right\rangle_{\mu_n} = \frac{\partial}{\partial B} \psi_n(\beta, B).$$

Pressure per particle:

$$\psi_n(\beta, B) = \frac{1}{n} \log Z_n(\beta, B).$$

Internal energy:

$$U_n(\beta, B) = -\frac{1}{n} \sum_{(i,j) \in E} \left\langle \sigma_i \sigma_j \right\rangle_{\mu_n} = \frac{\partial}{\partial \beta} \psi_n(\beta, B).$$

Thermodynamic limits

Theorem 9. [DemMon 10, DGvdH 10] For all $0 \leq \beta < \infty, B \in \mathbb{R}$,

$$\lim_{n\to\infty} \psi_n(\beta, B) = \varphi(\beta, B).$$

Thermodynamic limits

Theorem 9. [DemMon 10, DGvdH 10] For all $0 \le \beta < \infty$, $B \in \mathbb{R}$,

$$\lim_{n\to\infty} \psi_n(\beta, B) = \varphi(\beta, B),$$

where $\varphi(\beta, B)$ equals for $B > 0$,

$$\frac{\mathbb{E}[D]}{2} \log\cosh(\beta) - \frac{\mathbb{E}[D]}{2}\mathbb{E}[\log(1 + \tanh(\beta)\tanh(h_1)\tanh(h_2))]$$
$$+\mathbb{E}\left[\log\left(\mathrm{e}^{B}\prod_{i=1}^{D}\{1+\tanh(\beta)\tanh(h_i)\} + \mathrm{e}^{-B}\prod_{i=1}^{D}\{1-\tanh(\beta)\tanh(h_i)\}\right)\right],$$

with $(h_i)_{i\ge 1}$ i.i.d. copies of fixed point $h^* = h^*(\beta, B)$ of distributional recursion

$$h^{(t+1)} \stackrel{d}{=} B + \sum_{i=1}^{D_t^\star - 1} \operatorname{atanh}(\tanh(\beta)\tanh(h_i^{(t)})),$$

and $h^{(0)} \equiv B$. Here $(D_t^\star)_{t\ge 1}$ are i.i.d. size-biased degrees.

Thermodynamic limits

Theorem 10. [Dem-Mon 10, DGvdH 10] For all $\beta \geq 0, B \neq 0$:

(a) Magnetization.

$$M(\beta, B) \equiv \lim_{n \to \infty} M_n(\beta, B) = \frac{\partial}{\partial B}\varphi(\beta, B).$$

(b) Internal energy.

$$U(\beta, B) \equiv \lim_{n \to \infty} U_n(\beta, B) = -\frac{\partial}{\partial \beta}\varphi(\beta, B).$$

(c) Susceptibility. Let susceptibility be

$$\chi_n(\beta, B) = \frac{1}{n} \sum_{(i,j) \in E_n} \left(\langle \sigma_i \sigma_j \rangle_{\mu_n} - \langle \sigma_i \rangle_{\mu_n} \langle \sigma_j \rangle_{\mu_n} \right) = \frac{\partial M_n}{\partial B}(\beta, B).$$

Then,

$$\chi(\beta, B) \equiv \lim_{n \to \infty} \chi_n(\beta, B) = \frac{\partial^2}{\partial B^2}\varphi(\beta, B).$$

Ising phase transition CM

Theorem 11. [Dem-Mon 10, DGvdH 10] Ising model on locally tree-like random graph has critical value

$$\beta_c = 0 \quad \text{for} \quad \tau \in (2,3),$$
$$\beta_c = \operatorname{atanh}(1/\nu) \quad \text{for} \quad \tau > 3,\ \nu > 1.$$

Here $\nu = \mathbb{E}[D(D-1)]/\mathbb{E}[D] < \infty$ when $\tau > 3$ or $\mathbb{E}[D^2] < \infty$.

$\triangleright$ Means that, for $\tau \in (2,3)$, tiny external effects (media?) can cause opinion population to flip.

Critical exponents CM

Theorem 12. [DGvdH 14] For (β, B) close to $(\beta_c, 0)$,

$$M(\beta_c, B) \sim B^{1/\boldsymbol{\delta}}, \qquad M(\beta, 0^+) \sim (\beta - \beta_c)^{\boldsymbol{\beta}},$$

where

$$(\boldsymbol{\delta}, \boldsymbol{\beta}) = (3, 1/2) \qquad \text{for } \tau > 3;$$

while

$$(\boldsymbol{\delta}, \boldsymbol{\beta}) = (\tau - 2, 1/(\tau - 3)) \qquad \text{for } \tau \in (3, 5).$$

Non-universal exponents!

Insight: local analysis

▷ Behavior Ising model is to large extent determined by local behavior around all points.

Vertices far away hardly influence spins.

▷ Configuration model is

locally tree-like.

Means that local neighborhood around a vertex is close to tree, i.e., no cycles. Make life much **easier.**

□ Thus, need to understand Ising model on a tree in detail. For example, β_c is same as critical value on random tree.

Local tree-structure CM

$\triangleright$ Forward neighbors of neighbors of U_1 are close to i.i.d. Also forward neighbors of forward neighbors have asymptotically same distribution...

$\triangleright$ Conclusion: Neighborhood looks like branching process with offspring distribution $D^\star - 1$ (except for root, which has offspring D.)

$$\mathbb{P}(D^\star - 1 = k) \approx \frac{(k+1)}{\sum_{i\in[n]} d_i} \sum_{i\in[n]} \mathbb{1}_{\{d_i = k+1\}} \xrightarrow{\mathbb{P}} \frac{(k+1)}{\mathbb{E}[D]} \mathbb{P}(D = k+1).$$

Proof thermodynamics

Proposition 1. [Dem-Mon 10, DGvdH 10] Let $B > 0$ and let $(D_t^\star)_{t\geq 1}$ be i.i.d. random variables. Consider sequence of random variables $(h^{(t)})_{t\geq 0}$ defined by $h^{(0)} \equiv B$ and, for $t \geq 0$, by

$$h^{(t+1)} \stackrel{d}{=} B + \sum_{i=1}^{D_t^\star - 1} \operatorname{atanh}(\tanh(\beta)\tanh(h_i^{(t)})),$$

Distributions $h^{(t)}$ are stochastically monotone and $h^{(t)}$ converges in distribution to unique fixed point h^* that is supported on $[0, \infty)$.

$\triangleright$ Tree computation!

Conclusions

▷ Networks useful way to view real-world phenomena:

competition and consensus reaching.

▷ Unexpected commonality networks:

scale free and small worlds.

▷ Random graph models:

> Used to explain properties of real-world networks and as benchmark for brain networks.

Lecture 3:

Complex Networks and the Brain

- ▷ Brain as a Complex Network;
- ▷ Ising model as abstract model for brain functionality.

Networks of the brain

Several levels:

▷ Neuronal level: 10^{11} vertices of average degree 10^4;

▷ Functional level: much smaller, modular structure...

What is meaning network?

Features:

▷ Short time scales: stochastic process on network (non-linear?);

▷ Long time scales: network is changed by functionality brain (learning, pruning,...);

▷ Strong dependence between different regions network.

Big question:

What is a good network model for brain functionality?

Random graphs in/and Brain

Kozma-Puljic (05):

There is dominant view that brains are not random and one should not use the term random graphs and networks for brains.

Without going into metaphysical debate, it can be safely assumed that brains, viewed either as complex deterministic machines or as random objects, can benefit from use of statistical methods in their characterization.

All models are wrong...

George Box (78):

Now it would be very remarkable if any system existing in the real world could be exactly represented by any simple model. However, cunningly chosen parsimonious models often do provide remarkably useful approximations....

For such a model there is no need to ask the question “Is the model true?". If "truth" is to be the "whole truth" the answer must be "No". The only question of interest is "Is the model illuminating and useful?".

Question: How to “cunningly” choose a model for brain network topology and functionality?

Empirical brain networks

Complex brain networks: graph theoretical analysis of structural and functional systems

Bullmore and Sporns, Nature Reviews (09):

Empirical brain networks consistently show following features:

- ▷ Small worlds;
- ▷ High clustering;
- ▷ Hub-like degree structure, with (possibly exponentially truncated) power-law degree sequences;
- ▷ Hubs have rich-club organisation;
- ▷ Modular structure;
- ▷ Long-range spatial connections occurring at low rate.

Networks change with age.

Why network analysis?

Complex brain networks: graph theoretical analysis of structural and functional systems

Bullmore and Sporns, Nature Reviews (09):

Emprirical analysis shows that network topology is affected in patients with mental disorders:

Disorders investigated include Alzheimer disease (AD) and schizophrenia:

- ▷ Small world nature diminished, suggesting loss of efficiency of brain functionality;
- ▷ Clustering affected by AD, most often lower.

Ising and fMRI

Ising-like dynamics in large-scale functional brain networks

Daniel Fraiman, Balenzuela, Foss and Chialvo (Phys Rev E 09):

Interesting comparison of 2D Ising model and

fMRI data of brain resting state.

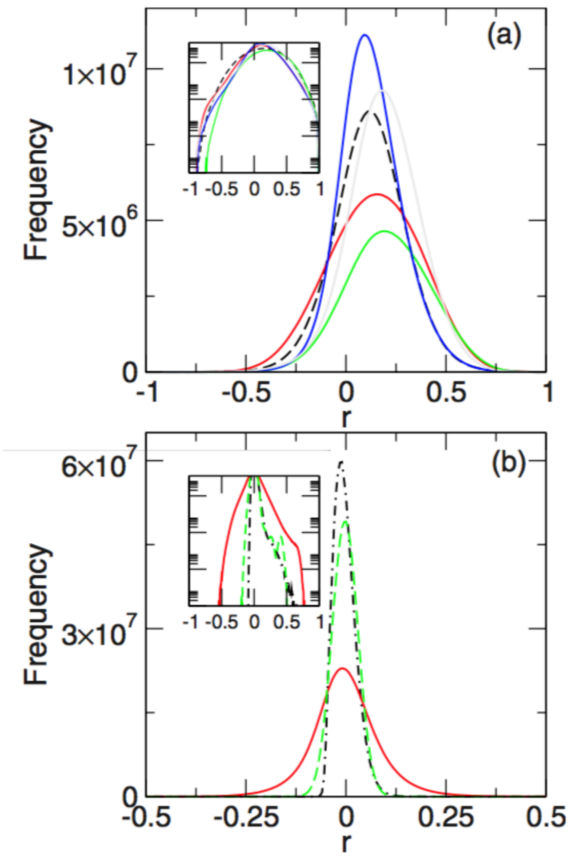
$64 \times 64 \times 49$ sites corresponding to voxels of dimension $3.4375 \times 3.4375 \times 3\ \mathrm{mm}^3$.

Compared measured correlations to correlations measured in dynamical Ising model at

critical temperature.

Reasonable comparison.

Ising-fMRI



(a)
Frequency
1×10^7
5×10^6
0
-1
-0.5
0
0.5
1
r
-1 -0.5 0 0.5 1
(b)
Frequency
6×10^7
3×10^7
0
-0.5
-0.25
0
0.25
0.5
r
-1 -0.5 0 0.5 1

Conclusions

▷ Networks useful way to view real-world phenomena:

friendship paradox and centrality.

▷ Unexpected commonality networks:

scale free and small worlds.

▷ Random graph models:

> Used to explain properties of real-world networks and as benchmark for brain networks.

▷ Graph theory useful tool for neuroscience.

▷ View brain functionality as stochastic process on brain network.