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# Design of a traction control system for the four-wheel driven race car IM01

*Bachelor Final Project*

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**Abstract**

Student team Automotive Technology InMotion is working on a high-powered electric four-wheel driven race car, the IM01. A common problem for race cars is that the torque they can deliver is higher than the torque that the tyres can transfer to the road surface. This results in problems when accelerating, for example, excessive wheel slip and a reduction of longitudinal acceleration. Therefore, in this report, a traction control system for the IM01 is designed.

A lot of research has already been done in the field of traction control and different concepts can be found in literature, this report looks into multiple traction control concepts. The advantages and disadvantages of these different concepts are reviewed and one concept is used to design the traction control system for the IM01. Also a way to determine the longitudinal vehicle velocity is found, this is done with a Correvit optical sensor, made by Kistler.

A quarter-car model is made to do simulations and to test the traction control system. Next to this quarter-car model, a tyre model is included, based on the Magic Formula. This tyre model is used to simulate tyre behaviour and to calculate tyre forces. The parameters for the Magic Formula model are based on the parameters of a Michelin LMP1 tyre, which are used in the concept design of the IM01. With this model, simulations have been executed to determine the maximum longitudinal tyre force, the maximum transmissible drive torque and the theoretical maximum acceleration.

A traction control system is designed that consists of a feed-back part, a feed-forward part and a peak friction estimator. In the feed-back part, the measured angular wheel speed is compared to an optimal angular wheel speed. This optimal angular wheel speed is derived from the optimal slip ratio which can be determined with the help of the Magic Formula tyre model. The feed-forward part uses the estimated longitudinal friction coefficient from the peak friction estimator, the gravitational force and the wheel radius to calculate a feed-forward torque. The peak friction estimator uses the vehicle mass, the gravitational force and the time derivative of the vehicle speed to estimate the longitudinal friction coefficient, this is then fed into the feed-forward controller.

Simulations have been executed for different conditions: without the traction control system, with only the feed-back part, with only the feed-forward part and with the full traction control system. The applied drive torques, the estimates longitudinal friction coefficients, the wheel and vehicle velocities and accelerations are then plotted for all of these situations. Also, the results for the peak friction estimator are plotted for a sudden decrease in the longitudinal friction coefficient.

A traction control system that does not use the wheel speed of a non-driven wheel as an input and is applicable to a four-wheel driven car has been designed. This traction control system is designed and validated through simulations and it is shown that it results in better performances than when no traction control system is used. A decrease of 40% in the $0 - 100km/h$ time is realised with the help of the traction control system. The peak friction estimator that is developed is able to estimate the longitudinal friction coefficient within approximately 1 second.

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# Chapter 1

# Introduction

Automotive Technology InMotion is a student team of Eindhoven University of Technology. InMotion's goal is to participate in the 24h of Le Mans, the biggest motorsport event in the world. The goal is to do this the so called Garage 56 class, this is an innovation class that has no imposed technical limitations except on safety. The car that is going to be built to compete in this race is the IM01, an electric driven four-wheel drive race car. In order to be able to achieve this ambitious goal, first a two-wheel driven electrical race car is built, the IM/e. This two-wheel driven race car will serve multiple purposes, such as testing new technologies, learning how to build a race car as a team and creating leverage to find new sponsors.

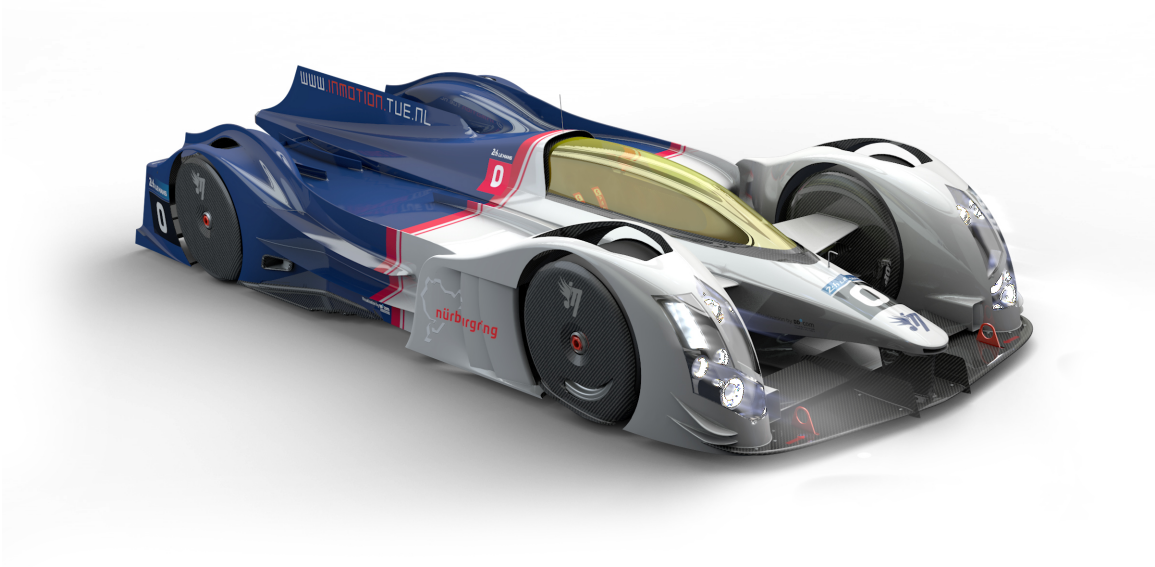



Figure 1.1: Render of the conceptual design of the IM01

## 1.1 Problem definition

Lately, a lot of research and development into electric vehicles (EV's) has been done. This is mainly to find a more environmentally friendly alternative for internal combustion engine vehicles (ICV's). Apart from the environmental advantages of EV's, the electric motors also bring another big advantage over internal combustion engines, the torque that is generated by the electric motors can be controlled much quicker and more precisely than that of internal combustion engines. Because this torque can be controlled this quickly and precisely, the stability of a vehicle can be improved greatly [2]. This is because a common problem with race cars is that the torque they can deliver is higher than the amount of torque the tyres can transfer to the road. When the applied torque to a tyre is higher than the maximum torque it can transfer, the wheel will start slipping, which will reduce in a loss of grip and as a result will lead to slower accelerations. In order to limit this wheel slip and to maximize the acceleration, a traction control system is often used, this is a controller which anticipates on how much torque can be applied and cuts down the applied torque when wheel slip occurs. Because both the IM/e as well as the IM01 are race cars and are built with the aim to be as fast as possible, a traction control system is needed. The objective of this report is to design such system.

Since the IM01 is driven a high powered race car, it can be assumed that traction control is required. A system is needed which can reduce the applied torque to the wheel when it slips too much. Also tyre data is used to make a first estimate of the amount of torque that can be transferred to the road surface. Because the IM01 is a four-wheel driven car, the traction control system has to be designed in a way it works for all four of those wheels. A problem that occurs is that the absolute vehicle speed has to be known in order for the traction control system to work. A way has to be found to determine this absolute vehicle speed without using the measurement of the wheel speed of the non-driven wheels. Another problem that occurs is that no road surface is the same, so the traction control system should be able to estimate the amount of grip that is available at any moment, this will be done by peak friction estimator.

*The purpose of this report is to design a traction control system that does not use the wheel speed of a non-driven wheel as an input and is applicable to a four-wheel driven car.*

## 1.2 Method of investigation

The traction control system is designed and tested with the help of a simplified model of a tyre and a quarter of a vehicle. In this report, only the traction control for one of the four wheels of a four-wheel driven car is considered. This is done because a race car usually has an ideal weight distribution between the front an the rear (50% front - 50% rear). Also, because generally, a race car has a very low centre of gravity, there is very little weight shift both longitudinal as lateral.

A few assumptions are made in order to get a simplified model of the reality, these are:

- A constant vertical force.
- Aerodynamic and rolling resistances are neglected.
- Tyre compression is neglected, so $r_w$ is constant.
- Only pure longitudinal slip is considered.

## 1.3 Outline

This report is built up out of six chapters. In chapter 2 a review of the existing literature about the subject will be given. It will explain the sign convention that is used, give an introduction

into the Magic Formula tyre model and will explain different traction control methods found in literature.

In chapter 3 the all the parts of the model that is used to design the traction control system is explained. First, the basic, quarter-car model is explained, this model describes the dynamics of the car and wheel. Second, the tyre model is explained, this is a model that is based on the Magic Formula tyre model and describes the characteristics of the tyre. Third, basic theoretical calculations are given about performances that should be possible and fourth, simulation results of the whole model are shown.

In chapter 4, the design of the traction controller is described and explained. All parts of the controller, namely: feed-back, feed-forward and the peak friction estimator are explained. Also, a block diagram of the whole traction control system is shown.

After that, chapter 5 shows the simulations of the traction control system. It shows the performance of the separate parts of the traction control system, it also shows what performance gains are achieved by the traction control system.

Finally, in chapter 6 conclusions will be drawn based on the results of the simulations. It will also give some recommendations for the implementation of the system and for future research.

# Chapter 2

# Literature review

The tyres of a car are the parts that transmit all the traction forces from the road surface to the car. Because of this, understanding tyre behaviour is necessary to be able to create a traction control system. A lot of research has been done in the field of tyre behaviour and a lot of different tyre modeling strategies are used, such as the brush tyre model, the Magic Formula tyre model and Van Rijks' tyre model. To be able to work with these tyre modeling strategies, a sign convention has to be introduced.

## 2.1 Sign convention

The sign convention that is used in this report is the sign convention by Besselink [1]. Figure 2.1 shows the sign convention with all the forces and moments acting on the contact point.

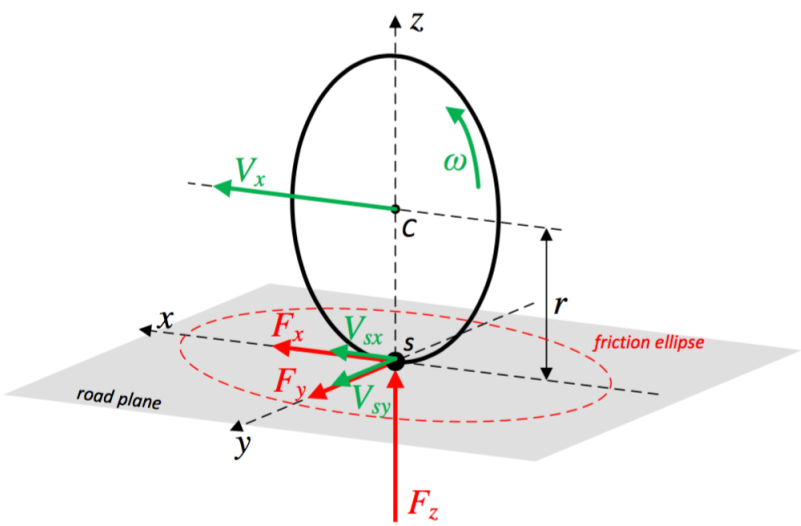



Figure 2.1: Tyre sign convention [1]

As can be seen in figure 2.1, the tyre axis-system is defined as follows:

- $z$: normal to road surface and upwards
- $x$: pointing forward, in the driving direction, through the wheel plane of symmetry
- plane $x, y$: parallel to road surface

The different forces acting on the tyre are defined as follows:

- $F_x$: longitudinal force (e.g. driving and braking)
- $F_y$: lateral force (e.g. steering), not considered here
- $F_z$: vertical or normal force (e.g. tyre compression)

The moments acting on the tyre are defined as follows:

- $M_z$: self aligning moment

Two different kinds of slip are defined, namely: lateral tyre slip $\alpha$ and longitudinal tyre slip $\kappa$. Lateral tyre slip $\alpha$, called side slip, is the ratio between the forward velocity, $V_x$, and the lateral velocity, $V_y$, at the contact point [7].

$$\alpha = \arctan\left(\frac{-V_{sy}}{V_x}\right) \tag{2.1}$$

The longitudinal wheel slip ($\kappa$) is the ratio between the speed of the contact point and the forward speed and is given by the following equation [7].

$$\kappa = -\frac{V_x - r_w\omega}{V_x} \tag{2.2}$$

In which $r_w$ is the effective wheel radius and $\omega$ is the angular wheel speed. From this equation it can be derived that $\kappa$ equals -1 when the wheels lock. When there is no wheel slip at all, so the wheel rolls freely, $\kappa$ is 0.

As stated in section 1.2, only pure longitudinal tyre slip is considered, so only $\kappa$ is used in this report.

## 2.2 Magic Formula tyre model

An often used way to model tyre behaviour is the Magic Formula tyre model, this is a semi-empirical tyre behaviour model which is used to model tyre force and moment characteristics in a steady-state situation. Since the first development of the model, multiple versions of the model have been used. The version used here and is given below is that by Besselink [1]. The Magic Formula tyre model is probably the most popular tyre model for vehicle handling simulations worldwide. The basic form of the Magic Formula tyre model is given by

$$F = D\sin(C\arctan((1-E)Bx + E\arctan(Bx))). \tag{2.3}$$

The Magic Formula tyre model for lateral forces is a function of both the vertical force $F_z$ as the lateral slip $\alpha$ as given below.

$$F_y = MF(\alpha, F_z) \tag{2.4}$$

The Magic Formula tyre model for longitudinal forces is a function of both the vertical force $F_z$ as the longitudinal slip $\kappa$ as given below.

$$F_x = MF(\kappa, F_z) \tag{2.5}$$

As stated in section 1.2, only pure longitudinal tyre slip is considered, so only equation 2.5 is used in this report.

The model used in this report is fully based on the basic form of the Magic Formula tyre model as given in equation 2.3. A further explanation of the model will be given in section 3.2.

## 2.3 Traction control

Traction control makes sure that the tyres of a car do not spin when accelerating, which is done by controlling the torque applied by the motor of the vehicle. This is necessary when the torque that is delivered to the wheel is higher than the maximum torque it can transfer to the road. Because then, the wheels will start slipping and the slip ratio gets too high, the transmissible tyre force will decline and so will the maximum transmissible drive torque. Because electric motors can deliver high torques at low speed and the IM01 is a four-wheel driven car that uses four electric motors, one for each wheel, the need for traction control in the IM01 is very high, this will be further discussed in section 3.4.

Figure 2.2 shows the longitudinal tyre force $F_x$ as a function of the longitudinal slip ratio $\kappa$ for a typical tyre. The friction coefficient $\mu$ is defined as

$$\mu_i = \frac{F_i}{F_z} \tag{2.6}$$

with $i$ equal to $x$ in the case of longitudinal slip and $i$ equal to $y$ in the case of lateral slip. As can be seen in figure 2.2, there is a slip ratio at which longitudinal tyre force, $F_x$, is at it's maximum, and so is the longitudinal friction coefficient, $\mu_x$, after that maximum, the friction coefficient decreases, and so does the tyre force. The traction control system attempts to make sure the tyre always operates as close to this maximum as possible to maximize the longitudinal acceleration, $a_x$.

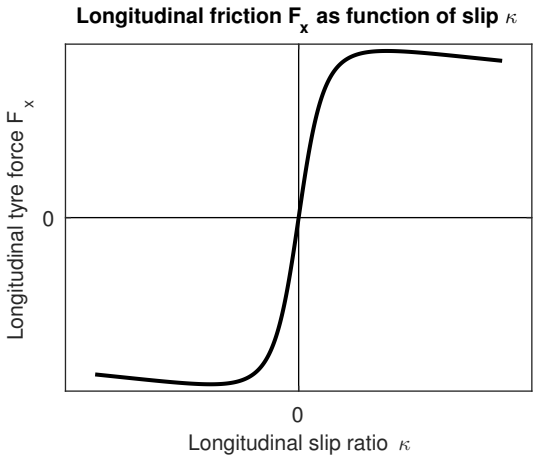



Figure 2.2: Longitudinal tyre force $F_x$ function of longitudinal slip ratio $\kappa$

A lot of research has already been done in different approaches of traction control systems. Three of them will be further explained in the following sections, namely: model following control, maximum transmissible torque estimation and slip ratio control. The advantages and disadvantages of all these approaches will be explained and an approach that is most suitable for the purpose of this report will be chosen.

### 2.3.1 Model following control

Model following control is an approach to control the slip ratio, it's a type of traction control that doesn't use any tyre data. Because of this, it is a relative simple type of traction control. Model following control will control the torque such that there will be no slip at all, $\kappa = 0$. Model following control uses parameters, most of which are constant. It uses the inertia of the wheel, the vehicle mass and the tyre radius to calculate a reference inertia, $J_{model}$ [2]. $J_{model}$ is then given by

$$J_{model} = J_{wheel} + m \cdot r_e^2 \tag{2.7}$$

What model following control then does is comparing the reference inertia $J_{model}$ to the vehicles inertia which is defined as:

$$J = J_{wheel} + m \cdot r_e^2 \cdot (1 - \kappa) \tag{2.8}$$

In equation 2.8, $\kappa$ is calculated by

$$\kappa = \frac{r_w \omega - V_x}{r_w \omega} \tag{2.9}$$

The difference between $J_{model}$ and $J$ is then fed back into a controller as an error which is given in equation 2.10.

$$e = J_{model} - J \tag{2.10}$$

The controller then uses this error to regulate the drive torque that is delivered to the wheel. The result is that this controller reduces the slip ratio to zero. As can be seen in figure 2.2, this is not the optimal slip ratio because the longitudinal friction is not at its maximum at that point. This is why model following control does not result in maximum longitudinal acceleration, $a_x$. Also model following control is a very rough approach, if we want to regulate the slip ratio better, a more precise approach is needed, that is why model following control is not used. A block diagram of a model following controller is given in figure 2.3.

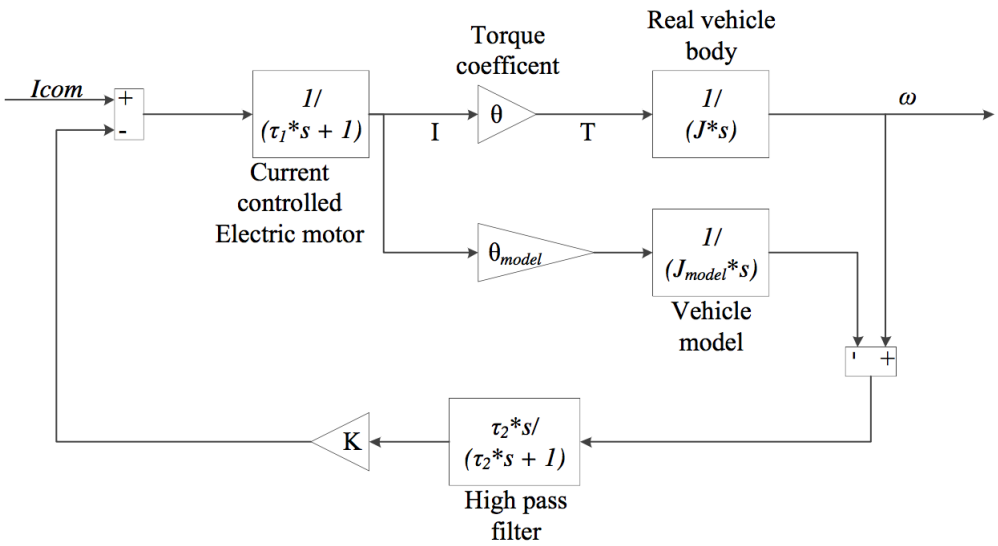



Figure 2.3: Block diagram of a model following controller [6]

### 2.3.2 Maximum transmissible torque estimation

Maximum Transmissible Torque Estimation (MTTE) [5] [8], is a traction control approach that does not use the actual vehicle speed $V_x$ as an input. This is ideal in the case of an four-wheel driven car, as is the IM01, because no additional sensors are needed to determine $V_x$. On top of that, MTTE doesn't require any tyre data. MTTE uses the applied torque to a wheel and the angular acceleration of that wheel to make an estimation of the maximum amount of torque that can be transmitted to the road.

In MTTE, the friction force of a tyre is given by

$$F_d = \frac{T}{r_w} - \frac{J_{wheel} \cdot \dot{V}_{wheel}}{r_w^2} \tag{2.11}$$

The maximum transmissible torque is then given by

$$T_{max} = \left( \frac{J_{wheel}}{\alpha \cdot m \cdot r_w^2} \right) \cdot r_w \cdot F_d \tag{2.12}$$

In which $\alpha$ is the relaxation factor of the tyre, which approximates the ratio of the acceleration of the wheel and the chassis, given by

$$\alpha = \frac{\dot{V}}{\dot{V}_{wheel}} = \frac{(F_d - F_{drag}) \cdot J_{wheel}}{(T_{max} - r_w \cdot F_d) \cdot r_w \cdot m} \tag{2.13}$$

The most important parameter to regulate the estimation of the maximum transmissible torque is $\alpha$. The value of $\alpha$ has to be chosen such that the performance is optimal. Parameter $\alpha$ can be used to balance the anti-slip and acceleration performances. When slip occurs, the wheel will accelerate too fast, $F_d$ will reduce and then $T_{max}$ will reduce as well.

### 2.3.3 Slip ratio control

Slip ratio control is a traction control method which regulates the angular wheel speed such that the actual slip ratio is as close to the optimal slip ratio as possible. It does this by comparing the actual measured wheel speed to an optimal angular wheel speed, $\omega_{opt}$ [2][6][3]. This optimal angular wheel speed is related to the optimal slip ratio, $\kappa_{opt}$ as given in equation: 2.14. This optimal slip ratio has to be found by using the Magic Formula tyre model as described in section 3.3.

$$\kappa_{opt} = -\frac{V_x - r_w \omega_{opt}}{V_x} \tag{2.14}$$

From this optimal slip ratio, the optimal angular wheel speed can be derived with equation 2.15.

$$\omega_{opt} = \frac{V_x(\kappa_{opt} + 1)}{r_w} \tag{2.15}$$

In equation 2.15, $V_x$ is the longitudinal velocity of the vehicle.

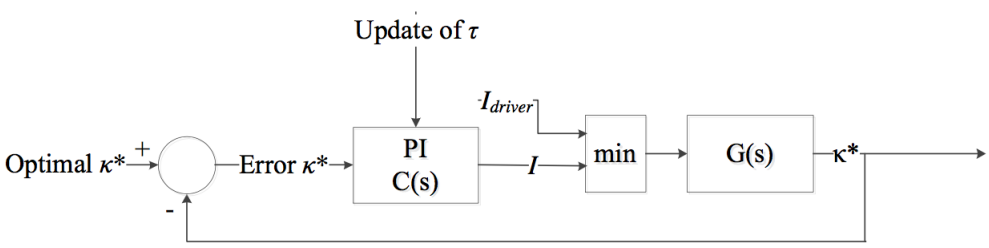



Figure 2.4: Block diagram of a slip ratio controller [6]

This slip ratio control method leads to the best results, as the wheel speed can be controlled best [6] [3]. That is why this method is used to design the traction control system. The design of the traction control system will be further explained in chapter 4.

### 2.3.4 Determining $V_x$

As the longitudinal speed of the vehicle, $V_x$ needs to be determined to determine $\omega_{opt}$, is has to measured in a good way. Usually, on two-wheel driven cars, $V_x$ is calculated from the angular wheel speed of the non-driven wheels, e.g. the front wheels in the case of a rear wheel-driven car. Because the IM01 is a four-wheel driven car, this is not possible because there simply are no non-driven wheels. That's why another way has to be found to determine $V_x$. Below, suggestions are given for determining $V_x$.

**GPS**

A way to determine $V_x$ is by using GPS. GPS can determine the location of a vehicle and by that, the velocity in any direction of a vehicle. So determining $V_x$ through GPS is possible, however, GPS has some downsides. Because we want to determine $V_x$ highly accurate with a high sampling frequency, GPS is not the ideal way. Also, GPS requires an unobstructed line of sight to four or more GPS satellites, which can't guaranteed at all time.

**Pitot tube**

A Pitot tube is a pressure measurement instrument, used to measure fluid flow velocity. It is widely used to determine the airspeed of an aircraft or the water speed of a boat, but is also used on high-end race cars. A downside of a Pitot tube is that it measures the airspeed of a car relative to the wind speed, so not its actual speed.

**Correvit sensor**

A Correvit optical sensor, by Kistler [4], is a sensor that can measure longitudinal and lateral vehicle dynamics in a slip-free way. Although this sensor is quite costly, it has a high level of accuracy, that's why it is used in for instance, F1, LMP1 and LMP2 race cars.

**Conclusion**

Because of the disadvantages of GPS and Pitot tubes, because the Correvit optical sensor has already proven itself in several professional racing series, and because it is a slip-free way of measuring $V_x$, it is recommended to use a Correvit sensor to measure and determine $V_x$ in the IM01.

# Chapter 3

# Modeling of the wheel and tyre

As explained in section 1.2, only a single wheel of the vehicle is evaluated in this model. As stated in chapter 2, all the traction forces from the road surface to the car are transmitted through the tyres, therefor tyre characteristics play a major role in the modeling of a traction control system. In this chapter, the model that is used to simulate the car and tyre behaviour will be explained. This model consists of two different parts, namely: the wheel dynamics model and the tyre forces model. First, in section 3.1, the dynamics of one wheel and a quarter of the car will be explained. Second, in section 3.2, the tyre model that is used to simulate the tyre forces is explained. Third, in section 3.3, some basic, theoretical calculations based on both the wheel dynamics model as on the tyre model are given. Last, in section 3.4, simulation results of the model are given.

## 3.1 Wheel dynamics

The dynamics of one wheel of the car will be modelled for the application of a drive torque applied by the motor to the wheel. Because only a quarter of the car is considered, a quarter of the total mass of the car is connected to this wheel. Figure 3.1 shows the sign convention that will be used in this report.

In figure, 3.1 $\omega$ is the angular wheel speed of the considered wheel in $[rad/s]$. The longitudinal velocity of the center of the wheel is represented as $V_x$ in $[m/s]$ in here. $T_d$ represents the drive torque in $[Nm]$ applied to the wheel. $F_x$ is the longitudinal tyre force in $[N]$ acting on the tyre. $F_z$ is the vertical tyre force in $[N]$ acting on the tyre. $I_w$ is the inertia of the wheel in $[kgm^2]$. The wheel radius is given by $r_w$ in $[m]$, and $m$ represents the mass of a quarter of the vehicle in $[kg]$.

The dynamics of the angular velocity of the wheel, $\omega$, and the mass of a quarter of the vehicle, $m$, can be described with the following equations of motion

$$I_w\dot{\omega} = T_d - F_x r_w \tag{3.1}$$

$$F_x = ma_x \tag{3.2}$$

Table 3.1 shows the values of the parameters used to describe the wheel dynamics in equations 3.1 and 3.2 which represent the parameters as they are set for the concept design of the IM01.

| | |
|---|---|
| $m$ | $300kg$ |
| $r_w$ | $0.356m$ |
| $I_w$ | $2.7kgm^2$ |

Table 3.1: Parameters for the wheel dynamics used in this model

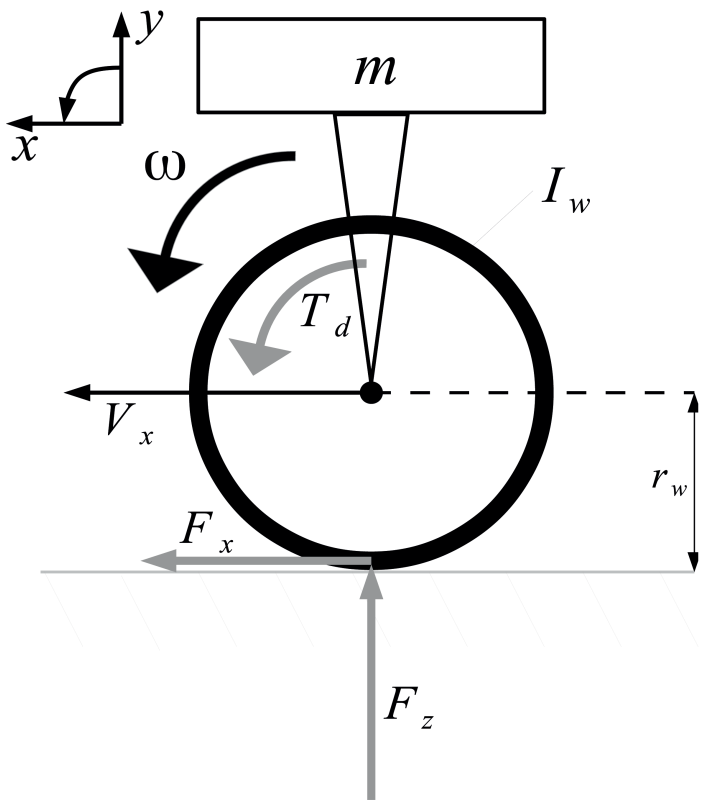



Figure 3.1: Dynamic wheel model

## 3.2 Tyre model

As stated in section 2.2, the tyre model that is used in this report is based on the Magic Formula tyre model, the version that is used here is the one given by Besselink [1].

### 3.2.1 Longitudinal tyre forces

For convenience reasons, only longitudinal tyre forces are taken into account in this report, as stated in section 1.2. The Magic Formula calculates the longitudinal tyre force $F_x$ as a function of $F_z$ and $\kappa$, the vertical force and the longitudinal slip respectively. The value for $F_z$ is based on a quarter of the mass of the vehicle as stated in section 3.1, it is then given by: $F_z = 300 \cdot 9.81 = 2943N$.

$$F_x = MF(\kappa, F_z) \tag{3.3}$$

The Magic Formula is then given by:

$$F_x = D_x \sin[C_x \arctan\{(1 - E_x)B_x\kappa + E_x \arctan(B_x\kappa)\}] \tag{3.4}$$

in which

$$D_x = F_z\mu_x \tag{3.5}$$

$$\mu_x = p_{Dx1} + P_{Dx2}df_z \tag{3.6}$$

$$C_x = p_{Cx1} \tag{3.7}$$

$$E_x = p_{Ex1} + p_{Ex2}df_z \tag{3.8}$$

$$C_F\kappa = K_x = B_xC_xD_x = F_z(p_{Kx1} + p_{Kx2}df_z) \tag{3.9}$$

$$df_z = \frac{F_z - F_{z,nom}}{F_{z,nom}} \tag{3.10}$$

$B_x$ can then be calculated from $C_x$, $D_x$ and $K_x$ by:

$$B_x = \frac{K_x}{C_xD_x} \tag{3.11}$$

$\kappa$ as described by [7] is given in equation 2.2.

$$\kappa = -\frac{V_x - r_w\omega}{\max(|V_x|, V_{min})} \tag{3.12}$$

In equation 3.12 a minimum velocity $V_{min}$ is introduced in the definition of $\kappa$ to avoid a division by zero when the vehicle is standing still, so when $V_x = 0$ [1].

The model coefficients are usually determined by numerical optimisation, they are compared to measurements and are then tuned to minimise the difference between model and measurements. Table 3.2 shows the fitting parameters as used in this model which are based on the parameters of a type of Michelin LMP1 tyres which is used in the concept design of the IM01.

| | |
|---|---|
| $B_x$ | $9.5528N$ |
| $C_x$ | 1.6642 |
| $D_x$ | $5.7637 \cdot 10^3 N$ |
| $E_x$ | 0.7409 |
| $F_{z,nom}$ | $4414N$ |
| $\mu_x$ | 1.9584 |

Table 3.2: Fitting parameters of the Magic Formula tyre model

## 3.3 Theoretical calculations

In this section, the optimal slip ratio, theoretical calculations of the maximum transmissible tyre force, $F_{x,max}$, and drive torque, $T_{d,max}$, of the tyre and the maximum theoretical acceleration, $a_{max}$, will be determined.

The optimal slip ratio $\kappa_{opt}$ is found by differentiating equation 3.4 as given in equation 3.13 and solving it for $\frac{\partial F_x}{\partial \kappa} = 0$. In figure 3.2, $\kappa$ is plotted against $F_x$. The value of $\kappa$ corresponding to the maximum value of $F_x$ is then found, which is $\kappa_{opt}$.

$$\frac{\partial F_x}{\partial \kappa} = -\frac{C_xD_x\cos(C_x\arctan(E_x\arctan(B_x\kappa) - B_x\kappa(E_x-1))) \cdot \left(B_x(E_x-1) - \frac{B_xE_x}{B_x^2\kappa^2+1}\right)}{(E_x\arctan(B_x\kappa) - B_x\kappa(E_x-1))^2 + 1} \tag{3.13}$$

The value for $\kappa_{opt}$ in the case of pure longitudinal slip is then $\kappa_{opt} = 0.2269$.

The maximum transmissible tyre force, $F_{x,max}$, can then be found by simply filling in $\kappa_{opt}$ in equation 3.4, the maximum transmissible tyre force that is then found is: $F_{x,max} = 5763.7N$.

The maximum transmissible drive torque is then found by multiplying the maximum transmissible tyre force, $F_{x,max}$, with the wheel radius, $r_w$.

$$T_{d,max} = F_{x,max} \cdot r_w = 5763.7 \cdot 0.356 = 2051.9Nm \tag{3.14}$$

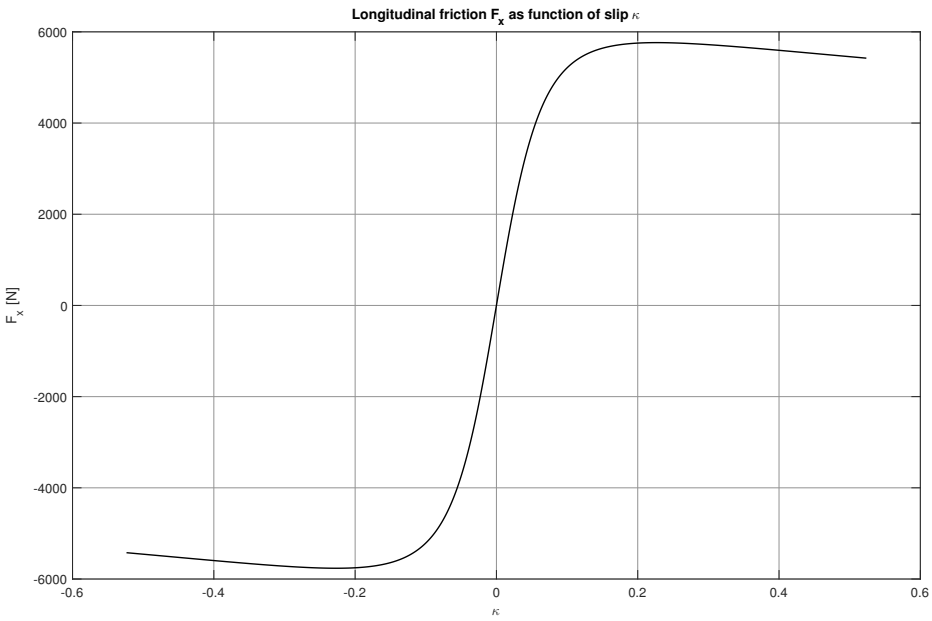



Figure 3.2: Longitudinal tyre force $F_x$ as function of the longitudinal slip ratio $\kappa$

By using equations 3.1, 3.2 and the value for $F_{x,max}$ that is found before, the theoretical maximum acceleration, $a_{max}$, can be calculated with equation 3.15.

$$a_{max} = \frac{F_{x,max} r_w^2}{m r_w^2 + I_w} = 17.9385 m/s^2 \tag{3.15}$$

## 3.4 Model simulations

Simulations have been done to show the need of a traction control system in a high-powered race car like the IM01. Three simulations have been done for different applied drive torques. The first one is with a drive torque of $1000Nm$, so a lot lower than $T_{d,max}$ as determined in section 3.3, the second is with a drive torque of $T_{d,max} = 2051.9Nm$, the third is with a drive torque of $3000Nm$, which is a lot higher than $T_{d,max}$. These simulations have been done to show that with a drive torque below $T_{d,max}$, performance is not at its maximum and with a drive torque above $T_{d,max}$, the wheels will start slipping because then $\omega \cdot r_w$ rises much faster than $V_x$. The results of these simulations can be seen in figure 3.3.

Second, the longitudinal accelerations for the same drive torques as before are plotted in figure 3.4. It shows that if the applied drive torque gets too high, the longitudinal acceleration $a_x$ is lower than when the applied drive torque is set at $T_{d,max}$.

Third, the angular wheel accelerations, $\dot{\omega}$, corresponding to the same drive torques as before are plotted in figure 3.4. As can be derived from figure 3.3, the angular velocity for a drive torque of $3000Nm$ is much higher than that for a drive torque of $T_{d,max} = 2051.9Nm$.

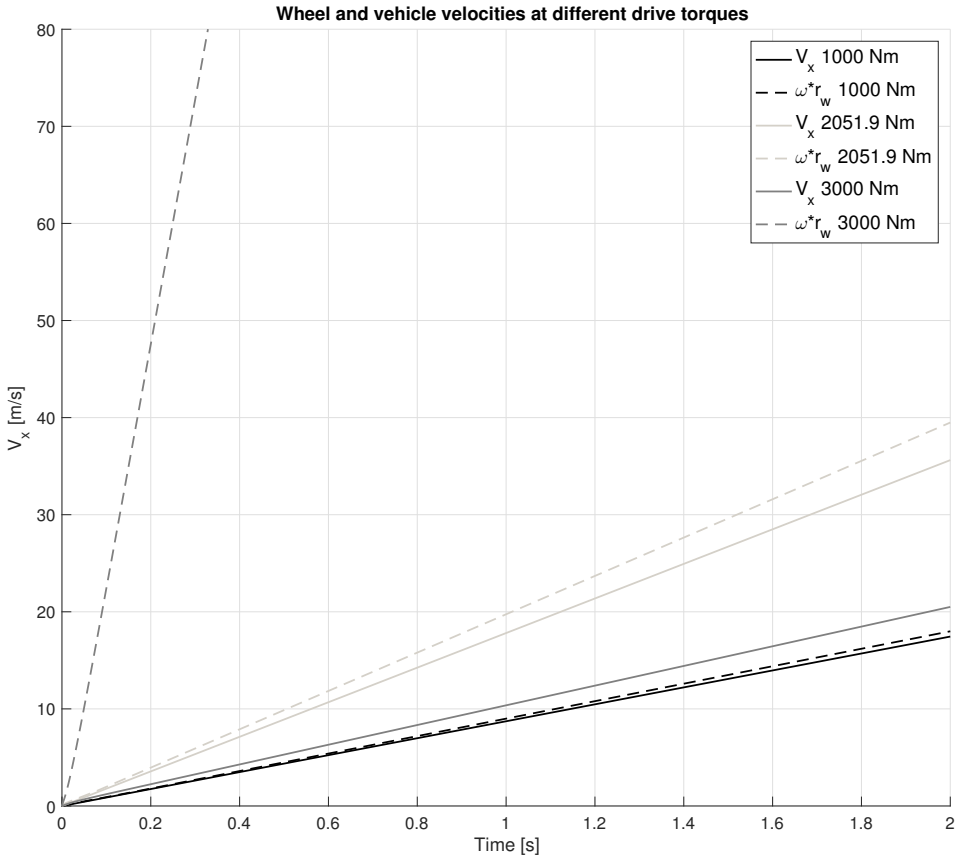
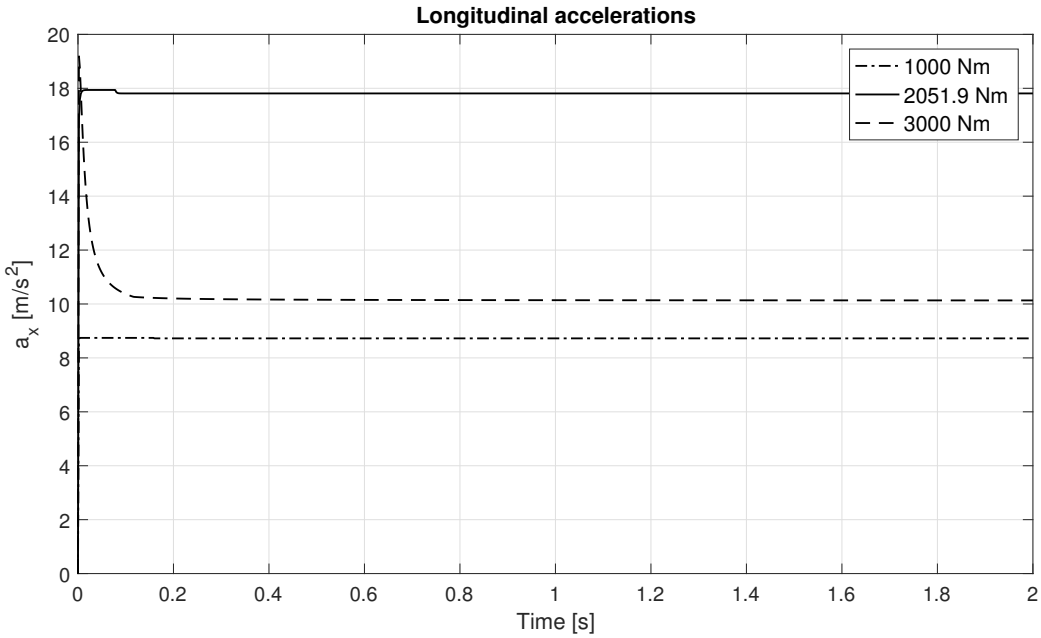



Figure 3.3: Vehicle and tyre velocities at different drive torques

Looking at these figures 3.3 and 3.4, the need for a traction control system in the case of the IM01 becomes very clear. When the applied drive torque is too high the wheels will start slipping as can be seen in figure 3.3, the longitudinal acceleration will drop as can be seen in figure 3.4. Figure 3.4 shows that the angular acceleration at an applied drive torque of $3000Nm$ way too high what also shows that excessive wheel slip occurs.



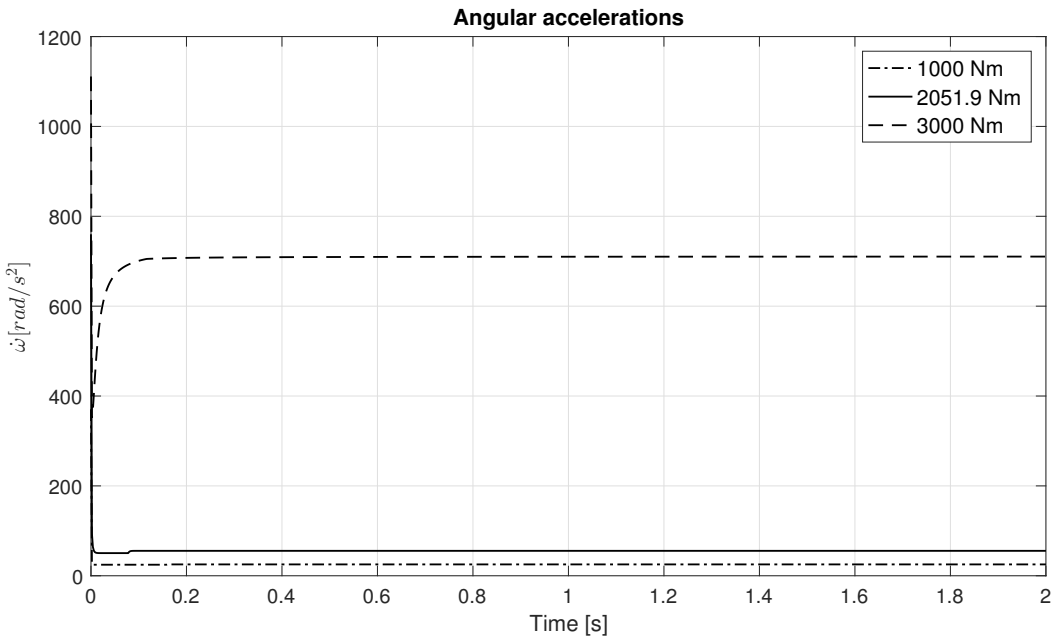




Figure 3.4: Longitudinal and angular accelerations at different drive torques

# Chapter 4

# Traction control design

The traction control system is designed to regulate the torque delivered to the wheels in a way such that traction is maximized. This controller consists of three parts, namely a feed-back loop, a feed-forward loop and the peak friction estimator. These will be discussed separately in the following sections. The concept of the controller system is illustrated in figure 4.1.

The block diagram as shown in figure 4.1 consists of several block who all represent a part of the traction control system. The peak friction estimator uses the wheel speed signal, $u$, to estimate the value of $\mu_x$, this value of $\mu_x$ is then fed into the feed-forward controller together with vertical tyre force $F_z$. The tyre model uses both the wheel speed signal, $u$, as the vertical tyre force $F_z$ to determine an optimal angular wheel speed, $\omega_{opt}$, from this optimal angular wheel speed, an error signal is generated according to equation 4.1.

$$\omega_{error} = \omega_{opt} - \omega_{measured} \tag{4.1}$$

This error signal is fed into the feed-back controller. The two signals from both the feed-forward as the feed-back controllers are then added into a controller torque. Because the driver should always have full control over the car, only the minimum of the controller torque and the requested torque by the driver is used to make up the torque request that is put into the motor. This torque request is then applied to the wheel which generates a wheel speed signal and a measured angular wheel speed, $\omega_{measured}$ out of it, these are fed back into the peak friction estimator and into equation 4.1 respectively.

Results of all the separate parts of the traction control system and of the complete traction control system will be shown in chapter 5.

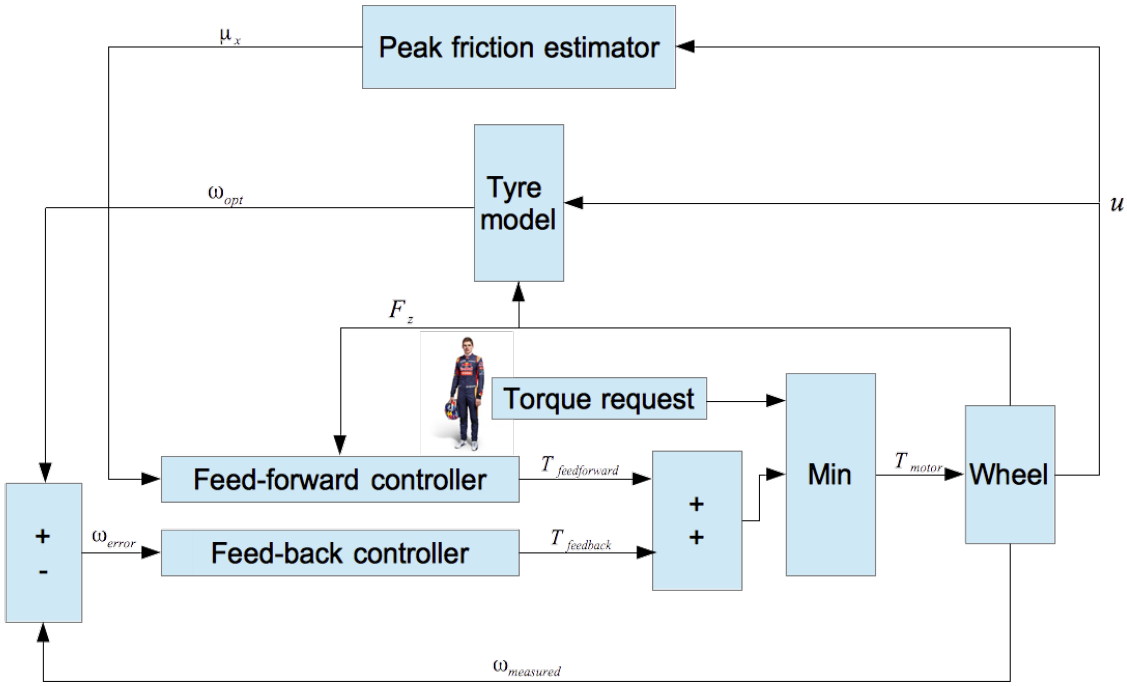



Figure 4.1: Block diagram of the traction control system based on [6].

## 4.1 Feed-back

The feed-back loop is constructed as explained in section 2.3.3. The optimal angular wheel speed, $\omega_{opt}$, serves as a set-point signal for the feed-back loop. That optimal angular wheel speed is compared with the measured angular wheel speed $\omega_{measured}$ and this is fed back into the feed-back controller as an error signal, $\omega_{error}$, see equation 4.1. This error signal is then controlled by a PI-controller to minimize it as quickly as possible in order to get the best performance possible.

## 4.2 Feed-forward

The feed-forward loop is constructed as can be seen in figure 4.1. This part of the controller uses the estimated longitudinal friction coefficient $\mu_x$ of the tyre estimated by the peak friction estimator, as explained in section 4.3. The vertical tyre force is assumed to be constant as given in equation 4.2:

$$F_z = 300 \cdot 9.81 = 2943N \tag{4.2}$$

The feed-forward torque is then calculated by:

$$T_{Feed-forward} = \mu_x \cdot F_z \cdot r_w \tag{4.3}$$

The feed-forward torque is used to give a torque which corresponds to the expected maximum value of the longitudinal tyre force, that is the force at which the tyre is expected to start slipping.

## 4.3 Peak friction estimator

The peak friction estimator is based on the method introduced by Y. Hori et al. [2] that is used to quickly estimate the value of $\mu_x$. This estimator is necessary because track conditions are never the same, so the amount of grip that is available won't always be the same. Because the vehicle speed $V_x$ can be measured directly, $\mu_x$ can be estimated by equation 4.4 [2]:

$$\mu_x = \frac{M}{F_z}\frac{dV_x}{dt} \tag{4.4}$$

In which $M$ is the vehicle mass and $F_z$ is given by equation 4.2. Results of the peak friction estimator will be shown in chapter 5.

# Chapter 5

# Simulation results

In this chapter, the simulations will be shown. The model as given in chapter 3 is used to simulate a situation in which the driver gives full throttle after 1 second. The drive torque that is requested is equal to $3000Nm$, this drive torque represents the expected maximum drive torque for the IM01. The drive torque requested by the driver is the same for all the simulations and is shown in figure 5.1.

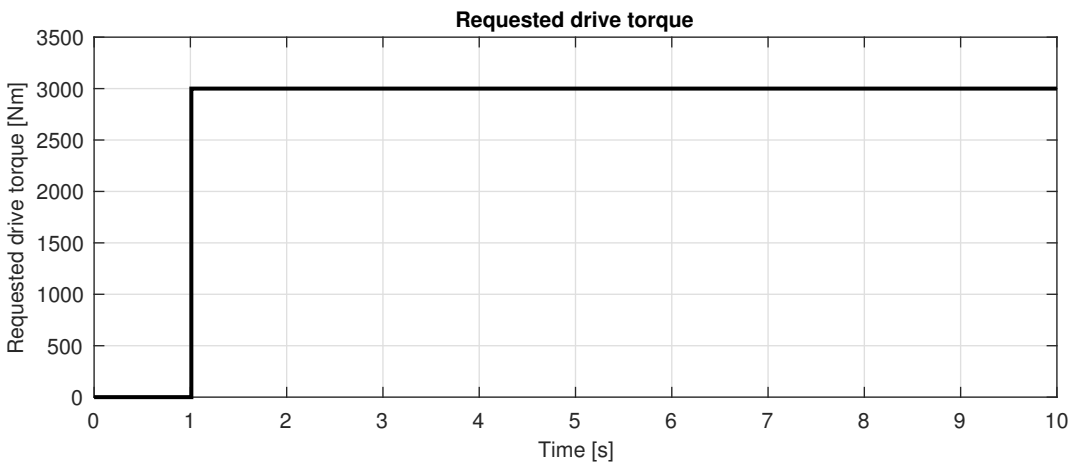



Figure 5.1: The requested drive torque for all simulations

First, the model without any form of traction control is simulated and the torque as plotted in figure 5.1 is requested. This is the situation in which the driver would requested the maximum amount of torque possible from the motors without letting go of the throttle. The results of this simulation are shown in figure 5.2.

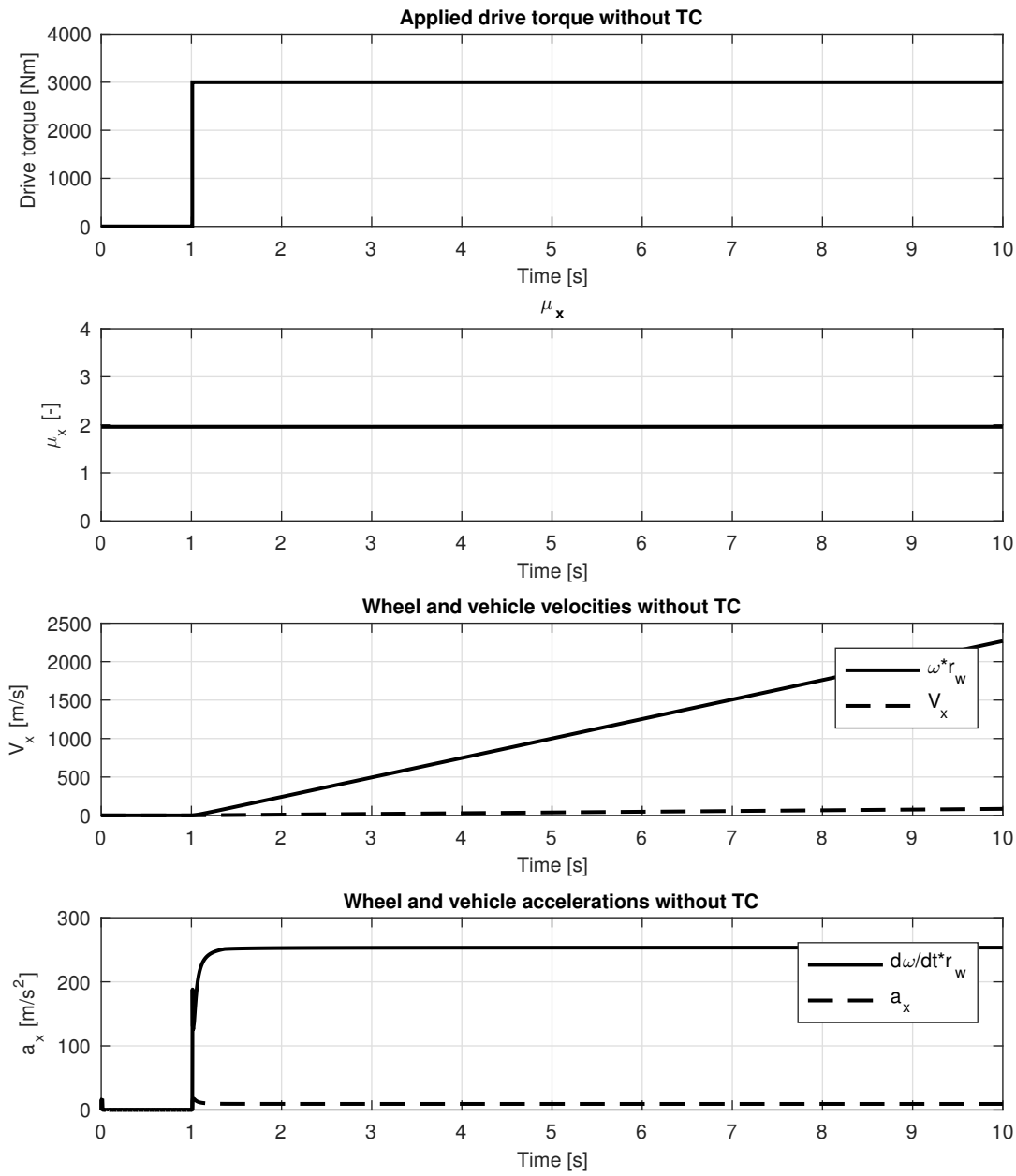



Figure 5.2: Simulation results without traction control

As can be seen in figure 5.2, the wheel velocity rises much faster than the vehicle velocity and the wheel acceleration is much higher than the vehicle acceleration, so the wheels will start spinning almost immediately after the torque request is drawn from the electric motors.

Second, the model with only the feed-forward loop implemented is simulated with, again the torque request as plotted in figure 5.1. So, in this simulation, no feed-back controller is used. The drive torque is only controlled by the $T_{Feed-forward}$ signal as explained and calculated in section 4.2. The results of this simulation are shown in figure 5.3.

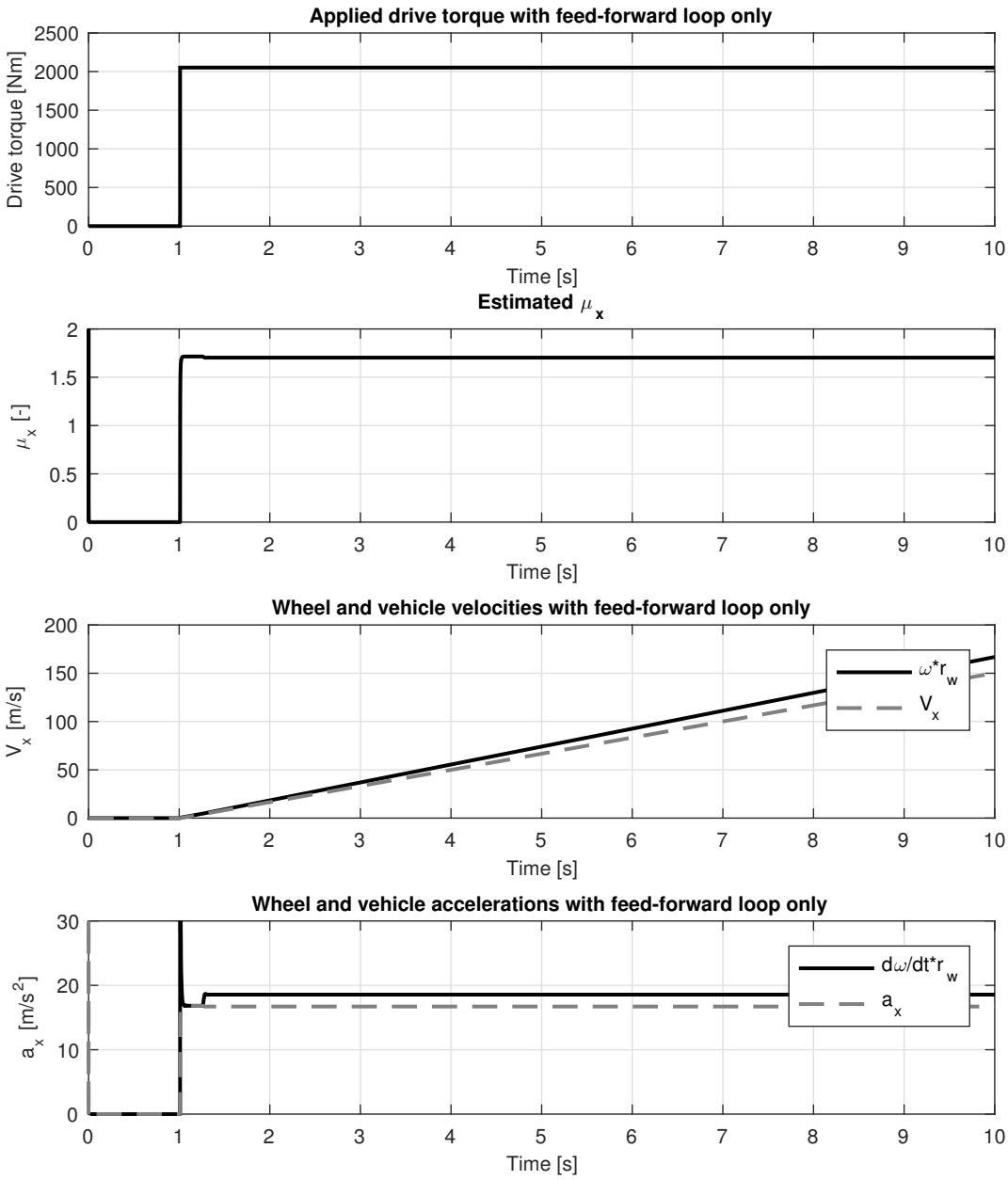



Figure 5.3: Simulation results with the feed-forward loop only

As can be seen in figure 5.3, the feed-forward controller cuts down the requested drive torque immediately and keeps the applied drive torque to the motor constant at $T_{Feed-forward}$ for the rest of the simulation.

Third, the model with only the feed-back loop implemented is simulated with, again, the torque request as plotted in figure 5.1. So, in this simulation, no feed-forward controller is used. The drive torque is only controlled by the $\omega_{error}$ signal and the PI-controller as explained in section 4.1. The results of this simulation are shown in figure 5.4.

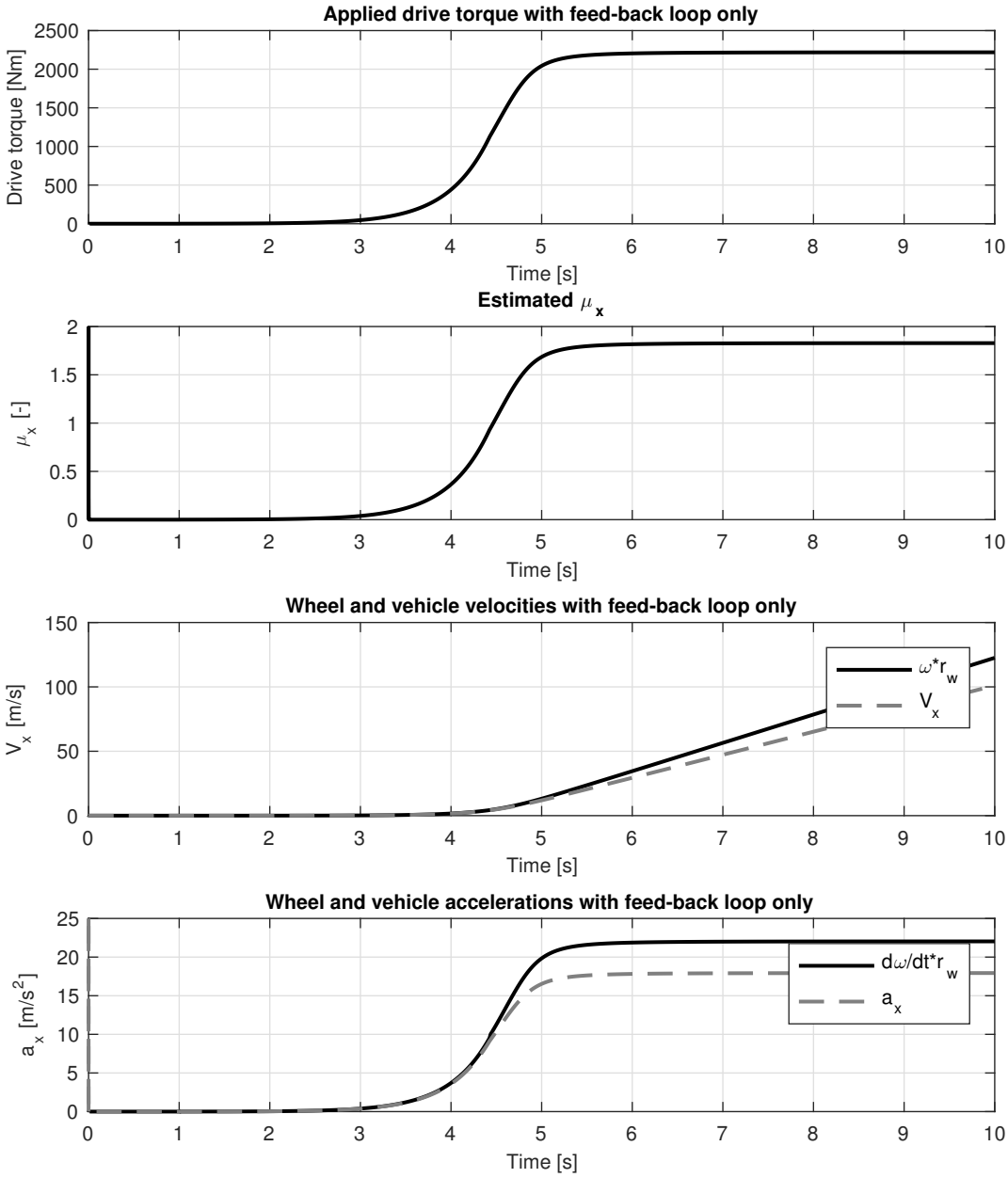



Figure 5.4: Simulation results with the feed-back loop only

As can be seen in figure 5.4, the feed-back controller cuts down the requested torque immediately, after that, the feed-back controller lets the applied drive torque increase slowly.

Fourth, the model with the full traction control system is simulated with, again, the torque request as plotted in figure 5.1. So, in this simulation, all the parts of the traction control system are implemented. The drive torque is both controlled by the feed-forward loop and the feed-back loop. The results of this simulation are shown in figure 5.5.

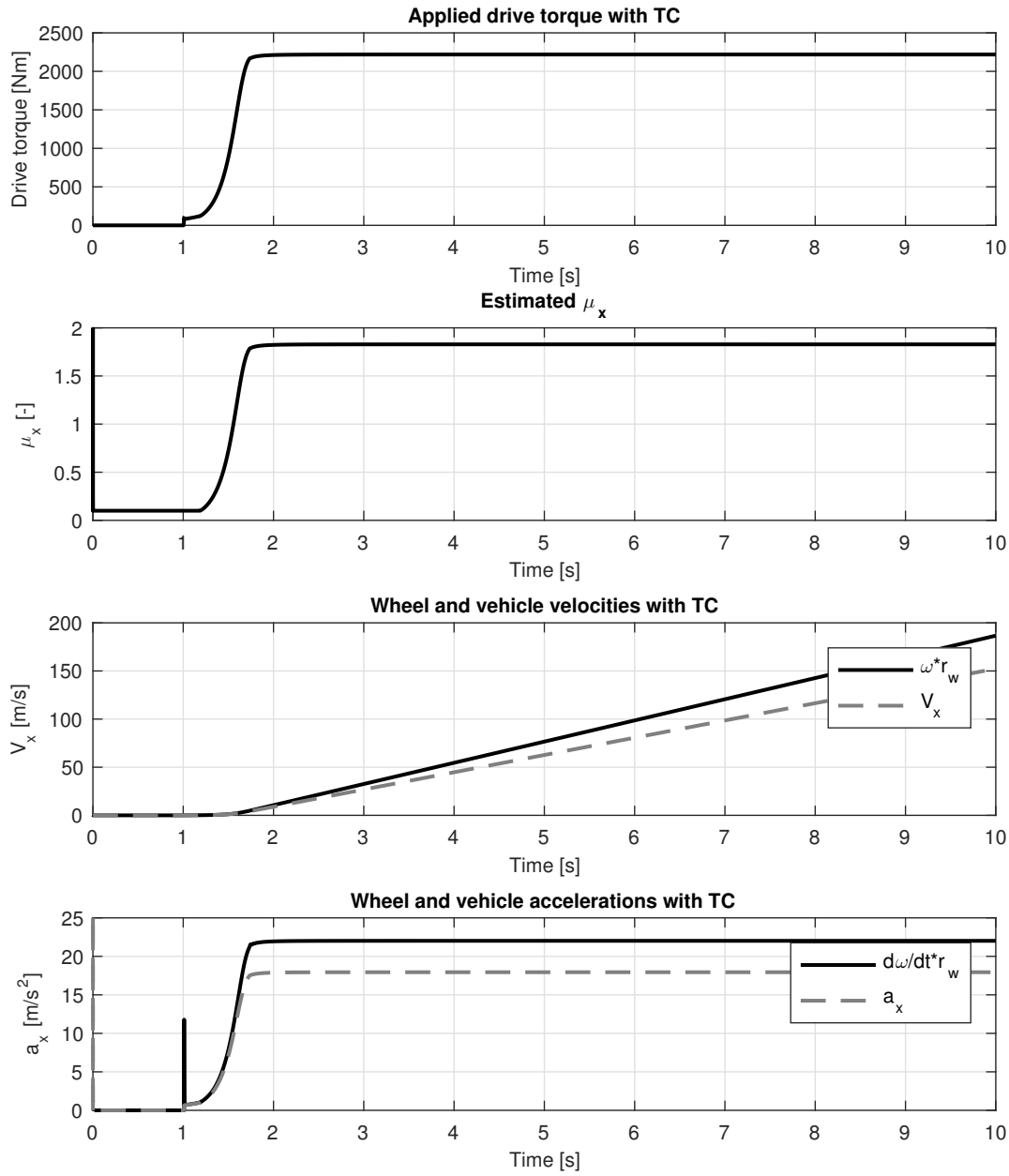



Figure 5.5: Simulation results with the full traction control system

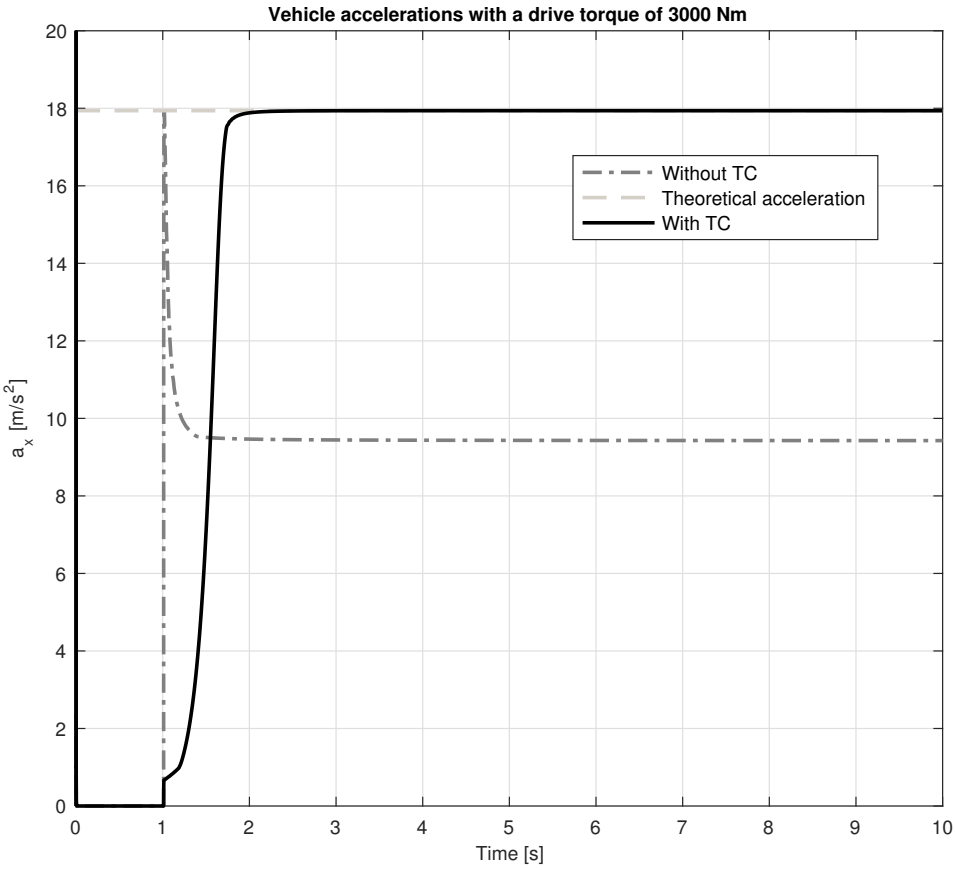



Figure 5.6: Vehicle accelerations with and without traction control and the theoretical maximum

As can be seen in figure 5.5, the traction control system lets the applied drive torque gradually rise to the value of $T_{Feed-forward}$. This results in a longitudinal acceleration which is very close to the theoretical maximum acceleration, the longitudinal vehicle accelerations are shown in more detail in figure 5.6.

In section 3.4, the theoretical maximum acceleration is calculated in equation 3.15 and is equal to: $a_{max} = 17.9385 m/s^2$. This theoretical maximum acceleration is plotted in figure 5.6 together with the acceleration with and without traction control for a requested drive torque of $3000 Nm$. As can be seen in figure 5.6, the vehicle acceleration that is realised by using the traction control system is almost equal to the theoretical maximum acceleration and is way higher than the vehicle acceleration without traction control, from this, and from comparing figure 5.2 and figure 5.5, it can be concluded that the designed traction control system works.

Also the $0 - 100 km/h$ times with and without traction control are determined, the results are shown in table 5.1. The $0 - 100 km/h$ is approximately 40 % higher with traction control than without.

| | |
|---|---|
| Without TC | $2.88s$ |
| With TC | $2.05s$ |

Table 5.1: $0 - 100km/h$ times with and without TC

In figure 5.7, the results of the $\mu_x$ estimator are shown, the estimated $\mu_x$ is plotted together with the estimated $\mu_x$ which is decreased with 1 after $2.5s$. The estimated $\mu_x$ becomes almost equal to the real value of $\mu_x$ as given in section 3.2.1. Also, $\mu_x$ which is decreased with 1 after $2.5s$ is estimated to the real value of $\mu_x$ minus 1. So it can be concluded that the $\mu_x$ estimator approximates the real value of $\mu_x$ and tunes its estimated value of $\mu_x$ after a decrease.

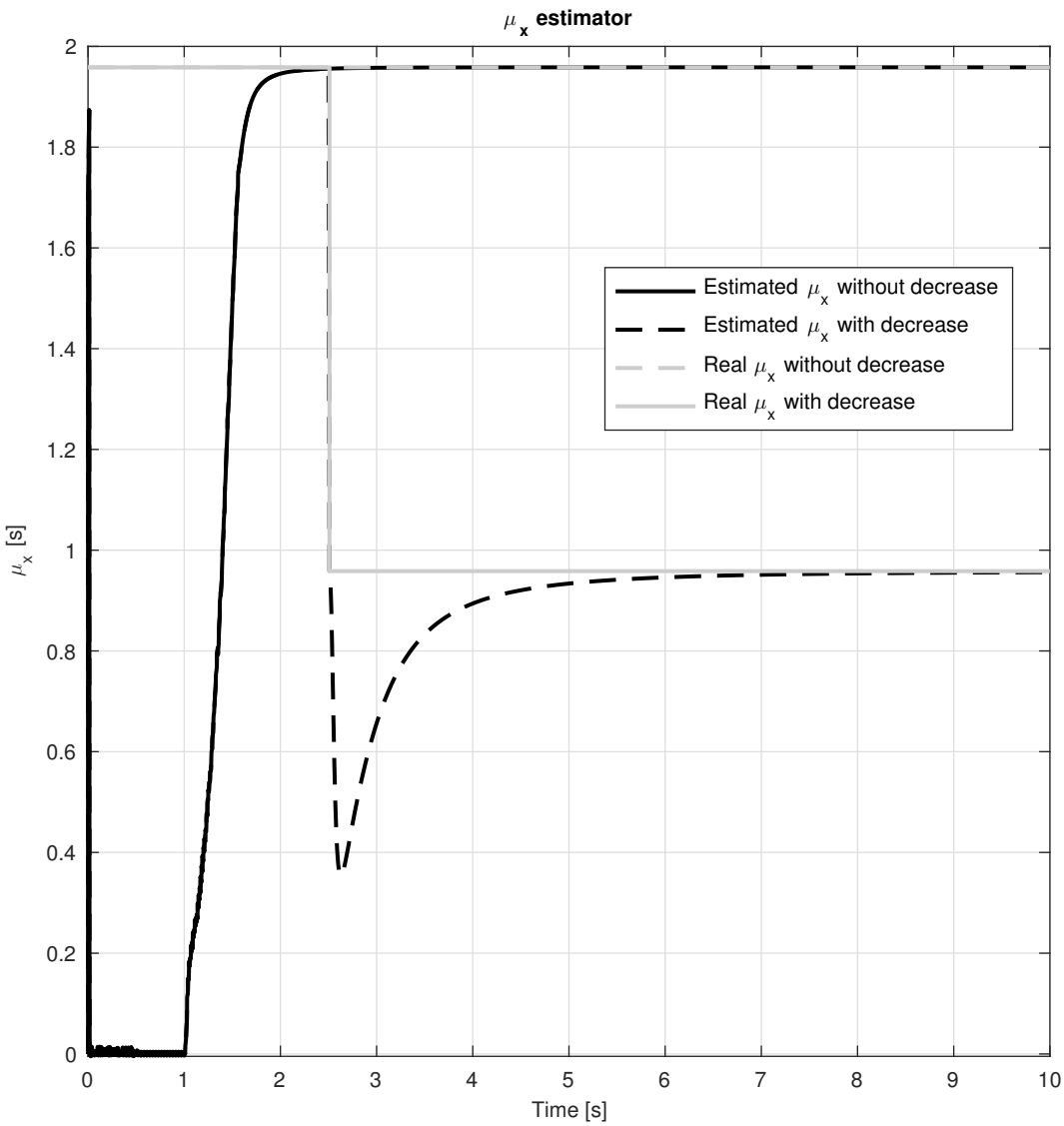



Figure 5.7: Results of the $\mu_x$ estimator

# Chapter 6

# Conclusions & recommendations

## 6.1 Conclusions

The objective of this report is to design a traction control system that does not use the wheel speed of a non-driven wheel as an input and is applicable to a four-wheel driven car.

In this report, a model is developed to simulate the dynamics of one wheel of the car for a drive torque applied by the motor to the wheel. A tyre model based on the Magic Formula tyre model is developed to simulate tyre behaviour and tyre forces. A traction control system is designed that does not use the wheel speed of a non-driven wheel as an input and that is applicable to a four-wheel driven race car as is the IM01. This traction control system is designed on one wheel only and is based on the slip ratio control approach. The traction control system consists of a feed-forward and a feed-back part and is then expanded with a peak friction estimator which estimates $\mu_x$. The validation of the traction control system is done with the help of the developed model.

Using the feed-forward loop only, the best performances can be achieved, however, variations in the road-surface and disturbances can not be controlled. That's why the feed-back loop is needed in the traction control system. Together, the feed-forward and feed-back loop can achieve better performances than without, but not as good as with the feed-forward loop only.

The traction control system that is designed and validated through simulations works and results in better performances than when no traction control system is used. The peak friction estimator that is developed works as well and is able to estimate the value of $\mu_x$.

## 6.2 Recommendations

Because some assumptions have been made in the dynamical model, as stated in section 1.2, the model can be improved by including the aerodynamic and rolling resistance for example. By doing this, the simulations will come closer to the real world and so, the design of the traction control system can be improved so it will deliver a higher level of performance. Also, tyre deformation can be taken into account what will lead to better results as well.

The peak friction estimator as it is designed for this report is based on the method introduced by Y. Hori et al. [2]. Other papers introduce other ways of estimating $\mu_x$ such as using a recursive least squares method, which should be investigated and will possibly lead to better results and to a faster working estimator.

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