

Stochastics: theory and applications		
Offered by	Department of Mathematics and Computer Science	
Language	English	
Primarily interesting for	All students with a strong interest and sufficient background in probability theory and statistics, but most relevant for students with background in Applied Mathematics	
Prerequisites	Strong interest and sufficient background in probability theory and statistics	
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Content and composition

The proposed package contains six elective courses in probability, statistics and measure theory, focusing on applications as well as on theory. These courses give a solid mathematical basis to understand modern probability theory and statistics, and explain how probability, statistics and stochastic operations research can be used in every day life. As such, they are an excellent preparation for Master's courses.

Stochastic models are used to describe broad classes of phenomena in physics, biology, chemistry, medicine, finance and computer-communication. This elective program covers a range of techniques and models that are used in these application areas, and explains how these models and techniques can be used effectively to gain insight in applications. Topics that are covered include simulation, probability models used in insurance and risk, extreme value theory, advanced statistical models and queueing systems. The course Measure, Integration and Probability Theory provides a fundamental basis for probability theory that will be useful in the entire master program.

Course code	Course name	Level classification
2WAG0	Measure, Integration and Probability Theory	3. Advanced
2WS70	Advanced Statistical Models	3. Advanced
2WS60	Extreme Values and Catastrophes	3. Advanced
2WB50	Stochastic Simulation	2. Intermediate
2WB40	Queueing Systems	3. Advanced
2DF30	Insurance and Credit Risk	3. Advanced

Students can choose any combination of three of these six courses to form a coherent elective program. All these courses require a solid knowledge of probability and statistics.



Course descriptions

2WAG0, Measure, Integration and Probability Theory

In his PhD thesis in 1902, Lebesgue introduced the concepts of measure and integration that now bear his name. These started a revolution in our thinking about size, volume, and integration.

A Riemann-integrable function is also integrable in the sense defined by Lebesgue, but the converse is not true. The class of Lebesgue-integrable functions is vastly larger than that of Riemann-integrable functions. And for the Lebesgue integral powerful convergence theorems are available that allow us, for instance, to exchange limit and integration, and therefore pass to the limit under the integral sign. Lebesgue's concepts



have revolutionized large parts of mathematics. Using these concepts, Probability Theory can be given a measuretheoretical foundation, which provides a very clean formalism and vastly simplifies many proofs. Measure theory is the basis for a large part of functional analysis, and opens the door to many abstract tools for the study of differential equations and variational calculus. In this course we introduce the basic building blocks of measure theory, define the integration of functions, and show how these concepts can be used in probability theory and functional analysis. The most important required prior knowledge is experience with the mathematical theorem-proof style of working. Further: open and closed sets, normed spaces, convergence of sequences, continuity of functions, and compactness. At the end of this course, the student has a thorough understanding of the concepts of Measure theory, Integration, and related topics in Probability Theory. (S)he can use these concepts to obtain answers to questions arising in Probability and Functional Analysis.

The following topics will be covered in this course:

- General measure theory: Sigma-algebra's, measures, Borel measures, regularity
- Lebesgue measure, relation with Riemann integral
- Important inequalities: Jensen, Cauchy-Schwarz, Minkowski
- Convergence theorems: Dominated Convergence, Monotone Convergence, Fatou's lemma
- Product measures and Fubini's theorem, convolution
- L^p-spaces: definition, completeness, relation between L^p-convergence and pointwise convergence
- Stochastic variables, distribution, expectation
- Independence, construction of countable sequences of independent stochastic variables
- Borel-Cantelli lemma's, Kolmogorov's 0-1 law
- Characteristic functions
- Law of large numbers and central limit theorem
- Conditional expectations

2WS70, Advanced Statistical Models

In many practical situations different types of data are being generated. A data set may contain continuous, discrete, and binary data representing certain properties of objects of interest. The applications we will consider are mostly from life sciences, which implies that we will often study measurements on living entities. Understanding the relationships between variables is often considered important. In the case of medical sciences it may help us understand health and disease. This course will discuss statistical methods and models for estimation of associations and for building nonlinear relationships between variables. The theory is applicable to any area of data science.

We expect a student to be familiar with probability theory and have a solid understanding of univariate statistics. Furthermore, students should have basic knowledge of linear statistical models. We assume students are familiar with calculus and linear algebra concepts such as partial derivatives, gradients, Taylor expansions, and matrix-vector multiplication.



The following topics will be covered in this course:

- The class of generalized linear models, including all the mathematics involved in fitting these as well as numerical issues;
- Drawing inference from generalized linear models, including all the mathematics these methods are based on;
- Choosing models, assessing goodness-of-fit, and assessing model complexity;
- Extending models to account for correlated data;
- Asymptotic statistics.

Next to theory, there is considerable attention to practical application; we will use real datasets, discuss how to extract meaningful conclusions in practical settings, understand the practical consequences of model choices, and discuss the reliability of our results.

2WS60, Extreme Values and Other Catastrophes

Unusual or extreme values can have devastating effects in real life. Think about floods, bankruptcy of insurers or a meltdown in a nuclear plant. As a result, we wish to make the probability of a disaster extremely small. The Dutch dykes are designed so as to withstand all floods except for once every 10,000 years. This raises the question how extreme events happen, and leads us to extreme value theory, a branch within probability and statistics that analyses extremal events and their probabilistic properties. In this course, you will learn the basics about rare events.

In this course, you learn what extreme values are and how to work with these. You learn about extreme value theory in the single-variate case and what limiting distributions can arise. You learn how to approximate probabilities of rare events, even outside the regimes where they have been observed. You also learn how to approximate probabilities of rare events when dealing with sums of independent random variables, such as occurring in insurance mathematics. For this, you learn the basics of large deviation theory.

Concerning prerequisites, we expect you to know the basics of probability theory and stochastic processes. In particular, you are expected to be able to work with independent random variables and sums of independent random variables.

The following topics will be covered in this course:

- Extreme value theory of independent and identically distributed random variables
- Extreme value distributions (Gumbel, Frechet and Weibull)
- Fisher-Tippett theorem for extreme value distributions
- Extreme value theory for partial sums, rare events and large deviations





2WB50, Stochastic Simulation

Many real-life processes are too difficult or too complex to analyze in an exact, theoretical way. The course discusses techniques to write computer simulations for a variety of real-life processes that exhibit a stochastic behavior, ranging from epidemic models, financial models, manufacturing networks, to models for interacting particle systems in physics. Since the course is very practical, it focusses mainly on the modeling aspect, and on developing simulations for these processes. The main goal of the course is that students not only learn and understand the required simulation techniques, but are also forced to actually write the simulation. Many of the lectures will be interactive, with the lecturer and the students writing a simulation together, step by step.



Elementary knowledge of probability theory and some basic experience in programming is required. The practical aspect of the course is also reflected in the way that the assessment takes place: students, working together in groups of two, will be given several assignments. Each assignment addresses a real-life problem. It is not only important that the students are capable of finding a suitable model for the problem, and writing a program that simulates this model, but it is also relevant that the student is able to interpret the outcomes, and use the simulation as a decision support tool to formulate recommendations on how the process might be improved.

Several programming techniques will be discussed, allowing the student to choose whichever suits him best. Although the programming languages R and Java are used during the lectures, the students are free to choose whichever programming language they prefer for their assignments.

The student learns to:

- Develop a stochastic model that describes the dynamic behavior of real-life processes
- Identify the states and the behavior of this model that are relevant for the development of a simulation of this particular model

- Write a program (in a programming language of his choice) that simulates the stochastic process and computes relevant performance measures

- Apply techniques to analyze (and, if necessary, improve) the accuracy of the simulation outcomes

Topics that are treated in this course are:

- Random number generators
- Generating random variables
- Monte-Carlo simulation
- Discrete-event simulation of several stochastic processes and queueing networks, like:
 - Random walks, birth-and-death processes
 - Epidemic models
 - Financial risk models
 - Interacting particle systems
 - Communication networks
 - Manufacturing systems and networks
- Input and output analysis (distribution fitting, warmup-interval, confidence intervals)

2WB40, Queueing Systems

Queueing phenomena are encountered in several real-life situations. Prominent examples are service counters, elevators and traffic networks, but queueing effects also arise in supply chains, production systems and communication networks. In this course you will learn basic mathematical models for analyzing congestion effects in terms of queue lengths and waiting times. You will also develop insight in applications of such approaches for improving the design and performance of service operations. Queueing models use basic probability concepts and stochastic processes (Poisson processes, Markov chains, birth-death processes, random walks) to describe and analyze congestion effects in terms of queue lengths and waiting times.





In this course we introduce the most common models in queueing theory, and explain how these models arise in various scenarios of interest. The focus is on mathematical techniques for deriving the stationary queue length distribution and waiting-time distribution, and calculating several specific performance measures. We also discuss how these methods and results can be applied for improving the efficiency or evaluating the performance of real-life service facilities in society, industry and technology. When taking the course, you are assumed to have followed an introductory course in probability theory, and have gained prior knowledge of concepts such as random variables, distribution functions, probability generating functions, Laplace-Stieltjes transforms, and stochastic processes (Poisson processes, discrete-time Markov chains, continuous-time Markov processes, birth-death processes, random walks, equilibrium distributions).

In the course you will learn several important models, insights and skills:

- Knowledge of the most common models in queueing theory, such as M/M/1, M/M/c, M/G/1 and G/M/1.
- Insight in several fundamental properties of queueing systems and the impact of various stochastic characteristics.
- Familiarity with basic approaches for analyzing queueing models (using probability generating functions and Laplace-Stieltjes transforms).
- Understanding how methods and results from queueing theory can be applied for improving the efficiency or evaluating the performance of real-life service facilities.

2DF30, Insurance and Credit Risk

In insurance and finance, randomness plays a major role. Insurance claim amounts and claim times are usually assumed to be random, and hence probability theory is needed to determine key performance measures like the probability that an insurance company gets ruined. Similarly, the stock market contains many random features, and a thorough knowledge of stochastic processes is needed to operate effectively in financial markets.



In this course, we discuss several models and methods which are relevant for the probabilistic analysis of insurance models, financial models and risk management. The focus of the course is on the understanding of models which are being used in practice for determining and quantifying the credit risk position of institutions like banks and insurance companies, as well as on acquiring knowledge and insight on theories and methods which play a fundamental role in analyzing stochastic financial and insurance models. We assume that you have followed introductory courses in calculus and probability theory, plus (in case you are not a math student) the first course (2DF20) of the coherent package Finance and Risk.

In this course, the following topics are treated:

1. Insurance risk:

- Collective risk models (insurance and reinsurance)
- Ruin theory (exact, asymptotic and approximate ruin probabilities)

2. Credit risk

- Rating agencies
- Mixed binomial model
- Vasicek one-factor model
- Value-at-Risk
- Economic Capital and Basel II

A special feature of the course is that, next to theory, there is considerable attention for practice:

- Active use of real data
- Actual implementation of all models
- Lecture notes on programming in R assignments, to be made in pairs, using real data and real implementations