

# Combinatorial optimization

Combinatorial optimization	
Offered by	Department of Mathematics
Language	English
Primarily interesting for	Bachelor Applied Mathematics students
Prerequisites	Required courses: Linear algebra, analysis, set theory and algebra Recommended courses: -
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## Content and composition

Combinatorial optimization is concerned with selecting the best from a finite set of discrete alternatives. A classical example is the travelling salesman problem, where the goal is to determine the shortest tour passing through a given set of cities. Even when it is clear that there is just a finite set of possible routes, it is not practically feasible to select the best by enumerating all possibilities. Problems such as this are common in science and business, arising as scheduling, routing, packing and rostering problems. The key problem of combinatorial optimization is to develop algorithms for solving such problems efficiently or for finding a reasonable approximation to the best possible solution in limited time. This package gives an introduction to several basic mathematical tools for analyzing and solving such combinatorial optimization problems: graph theory, linear and integer linear optimization, and a catalogue of standard algorithms together with an analysis of their efficiency.

Course code	Course name	Level classification
2WO20	Linear optimization	3. Advanced
2WF60	Graph theory and combinatorics	2. Deepening
2ILCO	Algorithms	3. Advanced

Note that also the combination of only the first two courses from this package, 2WO20 and 2WF60, is considered a coherent elective package (of 10 ECTS).

Students who would like to take the full package of (15 ECTS) are advised to take Graph theory and combinatorics (2WF60) already in the second year of the bachelor, as it contains some prerequisites for Algorithms (2ILCO) (for students of Applied Mathematics, the course 2WF60 can replace the course Datastructures (2IL50) as a prerequisite for 2ILCO).

## Course description

### Linear optimization (2WO20)

This course treats a key method for solving (discrete) optimization problems. Linear optimization is deciding the values of several unknown quantities, so as to maximize a given linear function of these unknowns, subject to given linear inequalities. There are outstanding algorithms for solving problems of this type. Many real-life problems can be cast as a linear optimization problem and solved as such. On the theoretical side, it is taught how to attain certainty about the optimality of solutions of linear optimization problems. This involves studying linear inequalities, the separation theorem for convex sets, Farkas' lemma, and the duality theorem for linear optimization. We describe the simplex method for solving linear optimization problems, and we prove that it works.



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Finally, we discuss two extensions of linear optimization: convex optimization and integer linear optimization. On the practical side, you will learn how to model real-world problems as linear optimization problems and to use existing solvers for evaluating the resulting models.

## **Graph theory and combinatorics (2WF60)**

In the combinatorics part subjects include counting problems, generating functions, recurrence relations and analysis of sorting algorithms. In the graph theory part, first structural properties of graphs are studied, and then several (algorithmic) graph problems are considered, such as Euler tour, Hamilton circuit, graph coloring, graph search, the shortest path problem, the minimum spanning tree problem, the maximum flow problem, and the maximum (weight) bipartite matching problem. We also touch upon the subject of computational complexity theory, by looking at possible (computer) representations of graphs and by analyzing the efficiency of the graph algorithms that are considered in this course. Graph theory and combinatorics is a level 2 course, which means that it can be taken in the second year of the bachelor applied mathematics (the only prerequisites are first year bachelor courses). The other courses in this elective package are level 3 courses.

## **Algorithms (2ILC0)**

This course consists of three parts. In the first part, three general techniques for solving optimization problems are studied: backtracking, dynamic programming, and greedy algorithms. The second part of the course deals with algorithms for optimization problems in graphs: shortest paths, maximum flows and matchings. In the third part of the course we study computational complexity theory by looking at NP-completeness, which investigates the limits of what is efficiently computable.



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This elective is about the design of business processes from an industrial-engineering point of view. This course focuses on business processes before the product launch (innovation, e.g. new product development) as well as after the product launch (e.g., production, service). Assignments focus on the (re)design of an organizational structure (e.g., sociotechnical redesign of an operational process in a production department), and the (re)design of a system (e.g. designing a performance measurement and feedback system for teams, or a decision support system for management). Ultimately, students are empowered to (re)design an innovation process and measure its effectiveness. This is an applied course with elements of challenge-based learning. It contains three group assignments in a production context as well as in an innovation context and therefore covers a broad spectrum of design problems. The design assignments are based on realistic case descriptions and are derived from actual projects.

## **Technology Forecasting (1ZK10)**

The development of a new technology is a process that typically requires a long time and depends on a multitude of technical and social factors. Firms who want to invest in an emerging technology need to evaluate the potential costs and benefits of such investments. If the firm invests early, it may have the option to profit from first-mover advantages, but it may also face the prospect of very long and costly research and development periods. If the firm invests late, the costs of adopting the technology are typically much lower, but the firm risks losing its competitive advantage to competitors. Technology forecasts therefore help the different stakeholders in an innovation system to assess whether, when, and how they should invest in a technology. The course covers the main analytical tools used to create these forecasts and the advantages and limitations of each type of tool. Course code, course name Provide a summary/description of the content, learning goals, etc.

## **Sustainability Perspectives on Product Innovation (1ZK20)**

New products increasingly have to present solutions to societal challenges such as climate change, environmental pollution, shortage of resources and excessive inequality, as the social and environmental sustainability of new products has become a new means for achieving competitive advantage for organizations. In addition, many (individuals in) organizations start recognizing their share of responsibility for the present societal challenges. They are thus in search of possible actions they can take to achieve more socially and environmentally sustainable and simultaneously profitable product innovations. In this course, you will learn which possibilities within and beyond the traditional product innovation process exist to increase the social, environmental and economic sustainability of the resulting new products and which actions they can take to make best use of these possibilities.