Analysis and Geometry elective package

Analysis and Geometry	
Offered by	Department of Mathematics and Computer Science
Language	English
Primarily interesting for	All students, but most relevant for students with background in Bachelor Applied Mathematics
Prerequisites	Required courses: Recommended courses:
Contact person	Title + initials and last name, e-mail address

Content and composition

Analysis is the branch of mathematics dealing with continuous quantities and the tools that we use to study them: functions, limits, differentiation, integration, measure theory, differential equations, and many others. Geometry is the study of space and of objects in it, and deals with shape, size, curvature, and the intrinsic properties of space.

Analysis and Geometry are active research areas, partly because of their importance to a wide range of applications. To give a few examples, the Navier-Stokes differential equations model the flow of liquids and gases; the equations describing quantum mechanics and quantum field theory are differential equations on curved space; and general relativity is formulated in terms of differential equations for the curvature of space itself.

The package is divided into three courses. Partial differential equations are differential equations in more than one variable, and describe many real-world phenomena, and in this course we study their basic properties. Measure Theory and Integration Theory provide the advanced concepts of size and integration that we use to study these equations and many other concepts; in addition, Measure Theory provides a rigorous foundation for the theory of probability. Finally, in the course on Tensor calculus and Differential geometry we study properties of curved space and curved surfaces.

This package is flexible enough not only to open the mathematics students access to modern mathematics (helping them choose a study/research direction for later), but it also provides the student in physics, chemistry or engineering the theoretical tools needed for understanding the fundamentals of quantum mechanics, statistical mechanics, continuum mechanics, theory of relativity, and medical imaging, for instance.

A note for non-mathematics students: these courses are aimed at students with a mathematical background, but can be followed by any student who is willing to brush up on the mathematics needed. As an indication of what provides a minimal mathematical background, you should be familiar with the concepts of theorem and proof, and with some basic notions of linear algebra (vectors, matrices, linear operators, eigenvalues and eigenvectors, diagonalization of matrices) and of calculus (differentiation, partial derivatives, Gauss theorem, integration by parts).
For Partial differential equations you also need to be familiar with ordinary differential equations. For Measure theory and Integration theory and for Tensor calculus and Differential geometry some basic elements of topology (open and closed sets, compactness, continuity) and of functional analysis (normed spaces, convergence in normed spaces) are needed.
Choosing two out of three of the courses also constitutes a coherent choice. For those students choosing this package, we strongly recommend also taking the course Functional Analysis (2WAF0).

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Course code	Course name	Level classification
2WA90	Partial differential equations	-
2WAG0	Measure, integration, and probability theory	-
2WAH0	Tensor calculus & differential geometry	-

Course description

Partial differential equations (2WA90)
For reliable answers to important questions such as:
- How is the weather going to be tomorrow or next week?
- What is the fate of the CO2 stored in the subsurface?
- How do nutrients reach (human) tissues?
-Has the coachwork in a new auto-design sufficient stiffness?

One needs to translate the processes into mathematical models. A large part of such models are partial different ial equations. This course presents the basic principles and techniques for understanding and analyzing such equations.

Students of non-mathematics majors are advised to follow this course in their third year.

Measure, integration, and probability theory (2WAG0)
Can you assign a surface area to every possible subset of the plane? Can you determine the integral of every possible function? Try to answer these questions, and you will open a Pandora's box called measure theory. But this curious by-product of innocent questions is foundational for at least two large areas of mathematics: Partial Differential Equations and Probability Theory. Without measure theory, many Partial Differential Equations could not even be formulated, let alone be shown to be well-posed. Measure theory really is the rigorous model for modern probability theory, and is essential for understanding general real-valued random variables and stochastic processes.

Tensor calculus & differential geometry (2WAH0)
A tensor is a multilinear extension of a linear map. Tensors are encountered in virtually any context in which linearization, a powerful generic technique used in mathematical modeling, plays a key role. Examples include continuum mechanics, electromagnetism, thermodynamics, relativity theory, and image analysis. The study of tensors is referred to as tensor calculus.

In many cases one considers tensor fields, i.e. tensor-valued functions defined on some base manifold, in practic e usually space or spacetime. One then enters into the realm of differential geometry, which simultaneously studies both the pointwise and neighbourhood interactions between the various tensorial quantities defined on this manifold.

The course Tensor calculus & differential geometry deemphasizes physical interpretations, but focuses instead on the generic mathematical machinery. Practical examples will illustrate the beauty of the theory.